REQUIREMENT FOR C-130 AIRCRAFT IN
THE INTRATHEATER KOREAN SCENARIO

THESIS

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Abstract

In an effort to provide a timely and reasonably accurate methodology for determining C-130 intratheater airlift requirements, this research concentrated on a rough-cut capacity approach using a straight forward linear programming spreadsheet model. To provide more detailed analysis, a more sophisticated linear program was investigated.

Specifically, the spreadsheet model calculated the minimum number of C-130s required to carry required cargo, passenger, and aeromedical loads based on user-defined daily requirements. For a given scenario, inputs include the daily requirements and the expected capacity for C-130 aircraft, trucks, and 22-car trains. Included in the capacity inputs are the number of daily cycles or trips expected from a given mode of transportation. The model is automatically formulated based on these inputs and is solved using a spreadsheet solver. Graphical results are provided. This spreadsheet model is analyzed for a 20 day period, but any planning horizon can be used with modifications. Since the spreadsheet does not perform a parametric analysis, the data used in the spreadsheet formulation was input into the LINDO solver in order to perform a parametric analysis. The parametric analysis was then imported into a spreadsheet and graphed.

I. Introduction

I.1. *General Issues*

The Air Force faces a decision to replace its aging fleet of C-130 "E" model aircraft with new C-130 "H2" and "J" models, institute service life extension programs (SLEP) for the E models, or some combination of both the previous options. Consequently, questions concerning the appropriate fleet size for the C-130 are being revisited. An overall requirements study will recommend the number of replacement aircraft the Air Force should procure based on a total required fleet size capable of meeting future contingencies. Air Combat Command (ACC) currently has responsibility for managing the C-130 fleet and ensuring this fleet has the capability of handling any contingency requiring intratheater airlift. The motivation for this thesis centers around the recent Department of Defense (DOD) policy to structure military forces to respond to two major regional contingencies (MRC) simultaneously. A classified Gulf War Air Power Study addressed the "western" MRC, but possesses no extensive analysis for the "eastern" MRC. In this light, HQ ACC needs an analysis of the "eastern" front to complete its C-130 requirements study. Based on the current world situation, this translates to an examination of a Korean scenario. (Stieven, 1995)

An idea of the organization and mission of intratheater forces can be found in Air Force Doctrine Document 30 which addresses airlift operations. The tasks of intratheater airlift are described as follows:

* Deploy and redeploy forces within the AOR (Area of Responsibility).

* Sustain deployed forces (both routine and combat sustainment).

* Deliver combat forces directly into battle.

* Force extraction from a combat environment.

* Conduct aeromedical evacuation operations.

* Augment strategic airlift forces when required.

* Perform non-lethal air power tasks such as foreign humanitarian assistance,

  leaflet drops, aerial spray, and fire fighting.

The use of intratheater airlift was instrumental in executing the wide flanking maneuver into Iraq during the Gulf War. This operation highlighted some primary characteristics of intratheater airlift, such as a quick reaction capability and the ability to operate from austere and unimproved landing zones (like sections of highway) (AFDD 30, 1995:18).

A review of several relatively recent historical examples illustrate the possible range in the size of potential deployments involving C-130 type aircraft. In Operation Desert Shield/Storm, the Air Force flew 46,500 intratheater sorties that moved 209,000 passengers and 300,000 tons of supplies. More than 145 C-130 aircraft were deployed in support of the operation. These airlift assets provided logistical support, aeromedical evacuation of the wounded, and battlefield mobility during the ground campaign. Approximately 500 sorties per day were flown during the ground campaign (Airpower,1991). Although not used to capacity, 11,250 patient beds were established through the deployment of 15 air transportable hospitals. Total patient visits amounted to

48,000 during the span of Operation Desert Shield/Storm (Airpower, 1991). The amount of personnel and materiel moved in the first month of the deployment gives an idea of the size of the initial deployment. Five fighter squadrons, an Airborne Warning and Control System (AWACS) contingent, and part of the 82nd Airborne Infantry Division were airlifted into the theater within the first five days. Within 35 days, the Air Force had deployed a fighter force into the Area of Operations (AO) that was the numerical equal of the Iraqi air force of 750 combat aircraft (Airpower, 1991). A review of the Korean order of battle for ground and air assets should give an indication of the size of any deployment that must be handled by the Air Force intratheater network.

Recently, the US Army made III Corps (Fort Hood, Texas) responsible for a Korean conflict as opposed to the I Corps. The primary units to deploy from III Corps would be the helicopter and artillery brigades according to one of the scenarios considered by the Army. These units would support troops already positioned in Korea and were chosen because they are relatively easy to airlift when compared to armored forces (Cole, 1996:386-7). The Assistant Secretary of Defense, Dr. Edward Warner, stated that the primary objective of US policy in the defense of the Korean peninsula is to hold the city of Seoul against a North Korean invasion, drive the opposing forces out of friendly territory, and ultimately achieve a decisive victory. He said the real challenge was to move in overwhelming airpower and "some land forces" (Warner, 1996).

Another recent example that highlights the low end of a possible contingency size can be found at the aerial port of Taszar, Hungary. This port operates as the US Army's premier logistics staging area for personnel and equipment going south to Bosnia-

Herzegovina in support of Operation Joint Endeavor and the North Atlantic Treaty Organization's (NATO) Implementation Force. Taszar has received more than 12,400 tons of cargo and 3,700 passengers from 11 December 1995 through 23 January 1996. This movement has consisted primarily of US Army support equipment and vehicles. (Ryder, 1996)

These operations constitute our most recent historical database to draw upon when conducting mobility requirements analysis. Our most recent experience with combat operations in Korea is, of course, not recent (early 1950s). Only large scale exercises, such as Exercise Team Spirit, give us an illustration of how military airlift operations in that region would be conducted.

The primary source of analytic results for ACC regarding the issue of intratheater airlift requirements is the classified Mobility Requirements Study (MRS) Vol III. The MRS was conducted from 1991-1993. This study was based on outputs from the TACWAR and MIDAS models. It calculated specific vehicle fleet requirements based on a static list of airlift movement requirements or with *time-phased force deployment data* (TPFDD). This data is translated into daily tonnage numbers once units arrive in theater. The system combining TACWAR and MIDAS is called SUMMIT and is a simulation designed for analysis of given fleet sizes. The Studies and Analysis Flight at HQ ACC is concerned about the value of analyzing the output from this study since many of the processes and assumptions of the associated models are currently unknown. They desire a generic model that is reasonably simple to understand, easy to set up, and simple

to run. Of course, the model must capture the relevant aspects of the intratheater airlift

system in order to provide a useful answer. (Stieven, 1995)

I.2.   *Specific Problem*

This research is guided by the primary assumption that the best model where the processes are readily comprehensible and portrayed with sufficient accuracy is through a deterministic approach. In particular, this research assumes that some amount of aggregation can be obtained from the cargo requirements as they are currently formatted in the time-phased force deployment data. The TPFDD data can then be used in a linear program to optimally determine the minimum required fleet to deliver the cargo. Obviously, some of the real world constraints on the airlift system will not be explicitly accounted for by such an aggregation. For example, facility throughput, materiel handling, and ramp space will be at best grossly modeled in an implicit manner. This can only be accounted for through the time dimension in the approach outlined in this thesis. However, the simplification of optimally determining fleet size has an important benefit for the analyst when compared to recent efforts at attempting this problem through sophisticated simulation approaches. When the Joint Staff conducted their Revised Intertheater Mobility Studies (RIMS), they required over 400 runs of the MIDAS model between October 1986 and April 1989 in order to determine airlift requirements. (Yang, 1995:4)

I.3. Definition of Research

The approach explores methods to capture the individual vehicle capacity of an intratheater transportation network to optimize these fleets. A basic linear program was investigated that modeled cargo, passenger, and medical evacuation requirements as resources (right hand sides). Vehicle capacities were modeled as technological coefficients (A matrix). Specifically, the research explored the use of a spreadsheet solver (Microsoft EXCEL©) to calculate the minimum number of C-130s needed to carry required cargo, passenger, and aeromedical loads based on user-defined daily requirements. A methodology is desired where the user need only enter the daily requirements for cargo (ton-miles), passengers, and aeromedical loads for a given scenario and the expected capacity for C-130 aircraft, trucks, and 22-car trains. The user should be able to individually enter daily capacities for the respective vehicles, i.e., capacities can be entered day by day. Included in the capacity inputs are the number of daily cycles or trips expected from a given mode of transportation. The model is automatically formulated based on these inputs. The model is solved using a spreadsheet solver and the results are provided in graphs. This spreadsheet model is formulated for a 20 day closure period since a larger planning horizon exceeds the number of variables the solver can handle. For larger closure periods, two or more formulations are entered and solved separately. The model results were verified by other solvers and used as inputs for a parametric analysis.

Another approach involved a set-partitioning formulation of a Korean airlift network to model the scenario. This model optimally determined the minimum number of C-130s required for the scenario. The set-partitioning model was based on determining the optimal number of routes which satisfy cargo and time window constraints imposed upon the nodes (onload/offload locations) of the network. The model is based upon the work found in the doctoral dissertation, "Network Optimization with Time Window Constrained Routing and Scheduling" by Fan Yang completed in August, 1995. A summary of this research is found at Appendix A.

I.4.   *Research Questions*

The primary research question asks, "What is the minimum number of C-130s required to achieve a desired closure profile for a given set of forces, support units, and resupply in a Korean scenario?" A secondary research issue is, "How do other modes of transport, i.e., truck and rail, affect the C-130 fleet size when they are introduced into the model?" One reason the primary question is timely is the type of current models available to airlift analysts in the Air Force. Current models are based upon simulations that mimic the movement of airlift and associated cargo based on certain rules. They accept data based on a fixed fleet size and address the question, "What is the closure profile for a given set of forces, support units, and resupply given a fixed set of transportation assets?" (Yang, 1995:9)

The data set for exploring this research question is notional. The actual data is classified, so this thesis identifies a methodology and model for the user who deals with classified data.

I.5.   *Scope*

An initial closure profile that is less than or equal to 20 days has been assumed. A force closure estimate should be used based on the appropriate operations plan (OPLAN). This force closure estimate is defined as the amount of elapsed time from the departure of the first aircraft from the load base to the arrival of the last aircraft at the offload base that completes the deployment of the initial combat force (USAF AMS, 1992:410-7). Sustainment airlift can begin before closure if the earliest units to arrive require such support, but normally sustainment missions are thought of as beginning after closure. It is assumed that the peak C-130 usage occurs within the closure time frame. It is further assumed that the time windows and cargo requirements will be known prior to the requirements study desired. The only cargo considered will be in the category of bulk cargo and oversize cargo. Bulk cargo is anything that will fit on a standard 463L pallet (88"x108"). Bulk cargo fits on all types of airlift aircraft. Oversize cargo is any single item that will not fit on a 463L pallet and exceeds the bulk dimensions listed above, but will fit on a C-130, C-141, or a C-5 aircraft. An example of oversize cargo would be a six-passenger truck or a HMMWV (Highly mobile military wheeled vehicle). Outsize cargo will not fit on a C-130 and is not considered here. An example of outsize cargo would be a tank or heavy artillery (USAF AMS, 1992:402-2). These requirements are assumed to be found in the TPFDD, which specifies the cargo and personnel requirement in the form of a data file. The TPFDD is generated with Joint Operation Planning and Execution System software as part of the deliberate planning process used in developing an OPLAN for a specific theater or contingency. These are classified documents and form the "starting

point" when a crisis action team actually begins to execute the deployment of military forces in a contingency or war (AFMAN 10-401, 1994) (USAF AMS,1992:405-1-405-10). The idea of uncertainty in this process is conveyed by the   well-known military adage that "no OPLAN has ever survived contact."   Since the TPFDD is our best guess for the scale of a possible contingency, the model will accept data from the TPFDD.

This study is further limited to generation of output data for a capacity model and a corresponding parametric analysis. A discussion of a more sophisticated approach that explicity models routing with time window constraints is presented, but no data from this model is analyzed.

Chapter II outlines the literature review conducted for this thesis and primarily concerns the exploration of vehicle routing problems and example formulations from the academic community and the military community.   Chapter III describes the methodology used to address the research questions discussed previously.   This chapter consists of descriptions of a spreadsheet application and a parametric analysis approach using a linear program formulation that allocates vehicles based on their associated capacities and specified transportation requirements. Chapter IV outlines a few examples to demonstrate the input and output of the model. Also included are brief descriptions of the data and worksheets employed in the analysis. An analysis of a potential scenario is given at the end of the chapter. The final chapter describes the assumptions and limitations of the model and suggests further research.

II.   Literature Review

The majority of the literature review concernes vehicle routing problems, as this study involved an airlift network. The spreadsheet model presented in Chapter III is based on a resources approach to linear programming and was motivated by modeling the daily requirements of airlift and the airlift capacity available to move the daily requirements. (Winston, 1994:71-112) This basic model was expanded to cover twenty periods with each period representing a single day.

A basic routing problem is easily stated as a set of nodes and arcs that must be serviced by a fleet of vehicles. In the basic problem, there are no restrictions on the order or timing in which the nodes must be serviced. The problem is to construct a low-cost, feasible set of routes. Each vehicle is assigned a single route. A route is defined as a sequence of locations that a vehicle visits and includes the service it provides. The routing of vehicles is primarily a spatial problem, since no temporal restrictions are placed on the problem (Bodin, 1983:79). An example of a basic routing problem is shown in Figure II-1. In this figure, the circles containing numbers and the rectangles containing depots represent the nodes of a network. The lines connecting the nodes are the arcs of the network. The entire set of nodes and arcs is called a graph. These arcs can be given direction, in which case the graph is called a di-graph. Figure II-1 does not

show any directional arcs (Bazaraa et. al.,1990:420).

Route 1: Depot A - 1 - 9 - 8 - Depot A
Route 2: Depot A - 5 - 2 - 3 - Depot A
Route 3: Depot B - 7 - 6 - 4 - Depot B
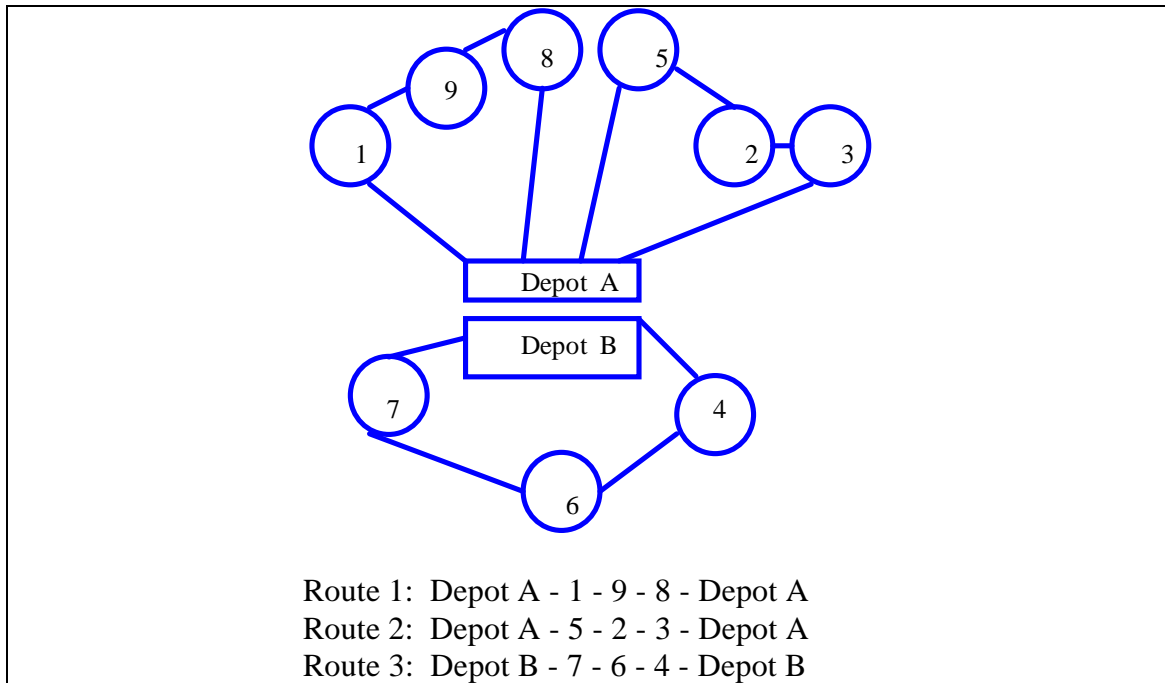
Figure II-1 Illustration of routes (Bodin, 1983:80)

A classic routing problem that demonstrates the mathematical simplicity of stating

such problems, along with the difficulties associated with their solutions, is the Traveling

Salesman Problem (TSP).   The TSP is based upon a network of nodes, arcs, and costs.

The graph could be named with the symbol "G", where G = [N, A, C] and N represents

the set of nodes, A represents the set of arcs, and C represents the set of costs for each

arc. The cost is either the distance between a node i and node j or the cost of moving

between these nodes and is represented with the symbol "c_{ij}".   The problem is to find the

minimum cost route starting from a given node and ending at that same node, visiting each

other node in the graph only once. It is called a Traveling Salesman Problem because the

problem can be thought of as a salesman that must visit each of the cities in a set and

return to his or her city of origin. Obviously, the salesmen will want to do this by traveling a minimum distance or at minimum cost (Bodin, 1983:82).

The TSP is NP-complete, which means that an algorithm that solves such a problem is not likely to be found whose solution contains a number of steps that can be described by a polynomial. All the algorithms that have been proposed encounter problems with storage and running time when used for networks with more than about 100 nodes. Proposed solution approaches are either hueristics, which give a feasible but not necessarily optimal answer, or algorithms which give exact optimal solutions. It is only practical to find an optimal solution for small problems of less than 100 nodes. One formulation of the TSP is: (Bodin, 1983:82-83)

$$x_{ij} = \begin{cases} 1 & \text{if arc } i\text{-}j \text{ in the tour} \\ 0 & \text{otherwise.} \end{cases}$$

$c_{ij} = +\infty$ for $i = 1, 2, ..., n$; and $i = j$;

$$\text{Minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = b_j = 1 \quad (j = 1,...,n) \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = a_i = 1 \quad (i = 1,...,n) \tag{2}$$

$$X = (x_{ij}) \in S \tag{3}$$

$x_{ij} = 0$ or 1 \quad $(i, j = 1, . . ., n)$

The symbol "S" represents a set that prohibits the situation shown in the following figure where subtours occur in the solution:

Constraint (1) makes sure each node has an arc going to it. Constraint (2) makes sure each node has an arc leaving it. We can see that these two constraints are satisfied by Figure II-2, but this obviously does not satisfy the problem definition of visiting each node and returning to the starting node. The following formulations are three ways to ensure subtours will not occur in the solution:

a) $S = \{(x_{ij}): \sum\limits_{i \in Q} \sum\limits_{j \notin Q} x_{ij} \geq 1$ for every nonempty proper subset Q of N $\}$;

b) $S = \{(x_{ij}): \sum\limits_{i \in R} \sum\limits_{j \in R} x_{ij} \leq |R| - 1$ for every nonempty subset R of $\{2, 3,\ldots, n\} \}$;

c) $S = \{(x_{ij}): y_i - y_j + nx_{ij} \leq n - 1$ for $2 \leq i \neq j \leq n$ for some real numbers $y_i\}$. (Bodin, 1983:83-84)
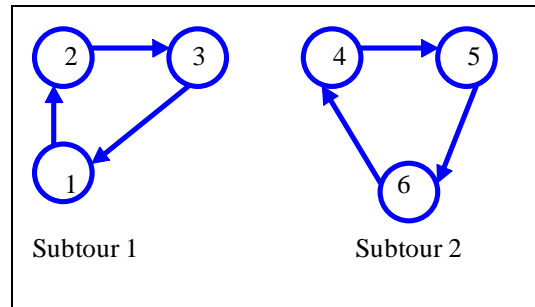

Figure II-2 Example of Subtours

A look at approaches to airplane scheduling gives one application of vehicle routing problems. Federal Express Corporation was among the first company to use a

computerized procedure for aircraft fleet scheduling. (Bodin, 1983:156) A timetable is

changed every four to five weeks to take into account changes in demand, new cities,

seasonality factors, and so forth. A matrix is used to make inputs. The matrix gives the

estimated package count between each pair of cities and this count is converted to a

percent of aircraft capacity. Each city is visited once. A package leaves its origin via

aircraft, arrives at Memphis, Tennessee and is then delivered to its destination city via

aircraft. Time windows specify the earliest time an aircraft can leave from a city of origin

and the latest time an aircraft can arrive at a destination with priority one packages.

Constraints limit the length of routes and the capacity of the aircraft. Two routing

problems are solved. One problem is for pickups and the other is for deliveries. (Bodin,

1983:156)

The algorithm used for the Federal Express Problem follows:

Assume we have N cities; city i has a known demand for packages to ship; and a time

window given by [t_i , $\bar{t}_i$] in which service is permitted.

Step 0: *definition of the savings*. The savings for cities i and j being on the same route (j

follows i): $s_{ij} = d_{0j} - d_{ij}$ where d_{0j} is the travel time from the depot to city j and d_{ij} is

the travel time from city i to city j. The savings are ordered: $s^{(1)} \geq s^{(2)} \geq \ldots \geq s^{(K)} \geq 0$.

Step 1: *iteration step*. Suppose for the first S iterations we have r partial routes

$R_1, R_2, \ldots, R_r$. A savings is then randomly selected by using a uniform distribution from

the next t savings on the list. This savings involves the joining of the partial route

beginning with city w to the partial route ending with city v. If the partial route

beginning with city w can be feasibly attached to the partial route ending with city v then

this attachment is made forming a new and longer partial route. Feasibility means that all time windows are satisfied, vehicle capacity (in terms of package count) is not exceeded, and the length of the tour is not too long. This step is repeated until the fleet size is met or all savings are exhausted.

Step 2: *merge step*. The new solution is merged into the present best known solution using the merge routine which uses an input of a set of feasible solutions. The first solution found is called the incumbent solution. As subsequent solutions are found, they are compared to the incumbent to determine if a reduced cost composite solution can be found. The rows of the two problems represent the legs of a route and the columns of the problems are the pairings from the two solutions.

Step 3: Steps 0, 1, and 2 are repeated until a satisfactory solution is found. (Bodin, 1983:167)

In response to a Congressionally Mandated Mobility Requirements Study (MRS), a method was developed in 1991 to determine the proper level and mix of lift assets necessary to support US power projection needs into the 21st century. The method used two linear program (LP) models. The first model was a multi-commodity network flow model based on scenario specific requirements to move units and their personnel and equipment. The second model minimized late delivery in order to assess the impact of a fixed but inadequate mobility mix. The method was called the Mobility Optimization Model (MOM) (Wing, 1991:1). The first model will be summarized here.

This method assumes a delivery schedule, a set of units to be moved, lift asset capacities, and cycle times of each asset. It determines the minimum cost mix of lift

assets that will close a force on time. The problem is constrained by the available load dates (ALDs) and the required delivery dates (RDDs) of the units to be moved. It is further constrained by throughput capabilities in the theater, a sustainment build-up policy in theater, and an initial inventory of lift assets. (Wing, 1991:3-4) Note: Cycle time is defined as the time that a lift asset requires to load, transit to the theater, offload, and return to the US for its next possible assignment. (Wing, 1991:5) All US bases were aggregated into a single source node and all terminal destinations were aggregated into a single sink node. (Wing, 1991:6) A flow constraint used in this formulation is given in Chapter V and an interesting objective function coefficient is formulated as follows:

Annualized life cycle costs were determined for all lift assets with the equation:

$$D(j) = \frac{PROCOST(j) + OPCOST(j) + E(LIFE)(j)}{E(LIFE)(j)} \tag{1}$$

- D(j) = annualized life cycle cost for lift asset j

- PROCOST(j) = procurement cost for lift asset j

- OPCOST(j) = annual operating cost of lift asset j

- E(LIFE)(j) = expected life of lift asset j

The annualized life cycle cost coefficient allowed the user to compare the costs for new, used, and leased lift options. The objective function equation is:

$$\text{Min } Z = \sum_{j} D(j) * \sum_{k} NM(j,k) + (PC * PREPO) \tag{2}$$

- D(j) = cost coefficient as calculated in equation (1)

- PC = Prepositioning cost factor

- PREPO = Number of ships required for prepositioning

- NM(j,k) = NEW(j,k) or new assets (j) required by the model. (Wing, 1991:14)

Another very promising approach came from a dissertation by Dr. Edward F. Yang of Washington University, Saint Louis, Missouri. This work described a new mobility analysis model called NETO that was formulated to address limitations of models currently used by Air Mobility Command in airlift analysis. The model consisted of a network optimization engine with time window constrained routing and scheduling that was based on an integer and combinatorial optimization methodology. The optimization engine is formulated as a Pickup and Delivery Vehicle Routing and Scheduling Problem with Time-Window Constraints (PDPTW) which is solved by a set-partitioning formulation, column generation and column elimination algorithm (SP-CGCE). The subproblem of the column generation is a Constrained Shortest Path Problem with pairing, precedence, capacity, and time window constraints which is solved by dynamic programming. (Yang, 1995:1-143)

III. Methodology

Since the ACC Studies and Analysis Flight conducts a significant portion of its analyses using "quick look" approaches, a spreadsheet model was investigated that was easy to set up and generated user-friendly output. Another desirable feature of this methodology was a "look through" logic where the decision maker could easily follow the logic and processes involved in the model and analysis. (Stieven, 1995) Microsoft EXCEL© was chosen since Air Combat Command analysts are familiar with this software and it is relatively easy to graph outputs from a solver solution. Unfortunately, the tradeoff for simplicity is a lack of sophistication in the model. The capacity approach does not consider the problem of routing vehicles and terminal facility capacities. Elements such as service times at the onload or offload locations and parking ramp space are not explicitly modeled. Routing of individual aircraft and the scheduling of aircrews are not directly modeled with this approach either. Since this is a linear programming approach, more decision variables and new constraints could be formulated to address the aforementioned limitations while not permitting violation of the new constraints. However, this would make the problem prohibitively large and too complex for a quick look study. These issues are discussed further in Chapter V.

It is important to note that the total cargo, passenger, and aeromedical requirements should be viewed as "chalks". A chalk is US Army terminology for a load of unit cargo and personnel that is planned to fit on *one* airlift mission. A US Army planner would ultimately determine how many and what type of chalks are required to

move a unit in a contingency. For example, the planner may determine his/her unit requires forty C-130 chalks, twelve C-141 chalks, and three C-5 chalks to deploy. This view of a chalk begins with dividing up unit equipment and personnel to fit load plans for a particular aircraft. An Air Force planner would view chalks in terms of weight, since it is convenient to think of airlift capacities in terms of weight. If the total requirement of cargo tonnage over 20 days is 112,500,000 pounds, we could divide this into chalks that would fit a particular vehicle. If our vehicle is a C-130 with a capacity of 25,000 pounds, we could divide our chalks into 25,000 pound increments and this would yield a total of 4,500 chalks. This translates into how many C-130s would be needed to move all the chalks of cargo for the entire 20 days if each C-130 made only one sortie in the entire planning horizon. Obviously, a more refined answer would be divide these chalks into daily requirements; otherwise, a possible solution would be to use 4,500 aircraft to haul everything in one day. The linear program will ideally divide the chalks among the various modes of travel that are modeled and divide them among each of the 20 days according to the capacity of each vehicle expected on a particular day. It should also be noted that the "chalk" concept assumes complete divisibility of the loads, a condition which does not always hold. It therefore understates vehicle requirements as some fractional loads must be flown. For instance, US Army planners would plan actual chalks for their unit equipment and personnel that may not completely use all the available capacity of a given vehicle.

Another aspect *not* explicitly modeled is the physical airlift network. This would be a detailed list of airbases and their characteristcs, such as ramp space, onload and

offload capability, and maintenance rates of servicing aircraft. The physical airlift network constrains the number of aircraft that can be handled at any one time, i.e., there are a limited number of possible air bases to handle aircraft and each base has a limited amount of parking space and cargo handling ability. In addition, planners put available dates and desired delivery dates on cargo and personnel they want moved during the development of the TPFDD. (AFM 10-401, 1994) This translates into a distributed demand for airlift over time. There will be surge periods and slack periods based on whatever military needs for cargo and personnel, i.e., military capability, were foreseen when the planners developed the TPFDD. A sobering comment that aptly demonstrates the potential problem of neglecting this aspect of the problem came from Dr. Abbe of the US Army Concepts Analysis Agency in her GDAS study, "What will NOT improve the deployment: (1) Adding more vehicles without matching build-up of facility capability [and] (2) Using alternate air and seaports without expansion of the supporting road and rail facilities." (Abbe, 1995:15)

This approach to the quick look study is one of daily vehicle capacity versus daily transportation requirements. The user must be able to "break out" the requirements data from the time-phased force deployment data (TPFDD). The spreadsheet requires that the first twenty days of requirements be broken into cargo tonnage per day, passenger counts per day, and aeromedical evacuation patient counts per day. This information can be obtained from the classified TPFDD document and broken out with various sorting codes available at the Air Mobility Command. (Shirley, 1995) The input page also allows the user to determine the capacities for C-130 aircraft, trucks, and several train vehicle types.

In addition, the capacity is multiplied by the number of trips expected from a particular vehicle for total daily capacity. The airlift requirements are linked to the right-hand-sides of a linear program formulation and the capacities are linked to the A matrix of coefficients in the linear program. It should be noted that ton-miles are used as the unit of measure for the cargo requirements in the study. This is a common metric used in airlift capability analysis and helps capture the overall distance factor. If two scenarios use the same exact amount of cargo in tons, but one demands airlift travel over an average of one hundred miles while the other requires travel of an average of three thousand miles to move the cargo, then using only a tonnage figure would not distinguish the scenarios. An example is the Congressionally Mandated Mobility Study of 1983 that arrived at a goal for all US airlift capability of 66 million ton-miles (mtm). (USAF AMS, 1992:430-1)

The number of vehicles used to meet the overall move requirements (cargo, passengers, and aeromedical patients) is represented in the spreadsheet model by the variables X1C_t, X1P_t, X1M_t, X2C_t, X2P_t, X2M_t, X3C_t, X3P_t, and X3M_t . The variable X1C_t is the number of C-130s used on day t to transport cargo or it could be any generic airlifter, depending upon the capacity entered on the input page of the spreadsheet (see Figure IV-1). The variable X2C_t is the number of trucks used on day t to transport cargo and the variable X3C_t is the number of 22-car trains used on day t to transport cargo. The capacity of each vehicle, i, to haul cargo is represented by the coefficient $a_{t(ic)}$. The capacity of each vehicle, i, to haul passengers is represented by the coefficient $a_{t(ip)}$. The capacity of each vehicle, i, to haul aeromedical (C-130 aircraft) or medical evacuation patients (trucks and rail vehicles) is represented by the coefficient $a_{t(im)}$.

The basic formulation is given below and in Figure III-1.

Variables: $X1C\_t$ Number of C-130s or generic airlifter required on day t loaded

with cargo.

$X1P\_t$ Number of C-130s or generic airlifter required on day t loaded with

passengers.

$X1M\_t$ Number of C-130s or generic airlifter required on day t loaded with

casualties.

$X2C\_t$ Number of trucks required on day t loaded with cargo.

$X2P\_t$ Number of trucks required on day t loaded with passengers.

$X2M\_t$ Number of trucks required on day t loaded with casualties.

$X3C\_t$ Number of 22-car trains required on day t loaded with cargo.

$X3P\_t$ Number of 22-car trains required on day t loaded with passengers.

$X3M\_t$ Number of 22-car trains required on day t loaded with casualties.

$$t = 1, \ldots, n$$

Xp Maximum number of airlift aircraft used in the planning period.

Xt Maximum number of trucks used in the planning period.

Xr Maximum number of 22-car trains used in the planning period.

w_p The objective function coefficient for Xp. This can be varied to

indicate the subjective priority of the airlift mode of transportation

in relation to other modes of transport. In other words, this is the

weighting factor for planes.

wₜ   The objective function coefficient for Xt. This can be varied to

            indicate the subjective priority of the truck mode of transportation

            in relation to the other modes of transport.

wᵣ   The objective coefficient for Xr. This can be varied to indicate the

            subjective priority of the rail mode of transportation in relation to

            the other modes of transport.

Constraint number (1) in Figure III-1 on page 26 ensures that the cargo

requirement is met. This means that enough vehicles must be assigned with sufficient

capacity to move the required cargo for a given day using the following formulation:

$$a_{t(1c)}(X1C\_t) + a_{t(2c)}(X2C\_t) + a_{t(3c)}(X3C\_t) \geq b_{tc} \; ; \; t = 1, \ldots, n$$

This formulation will form n rows where each row indicates a day's cargo activity. A

single row or constraint from this formulation is called a "day t cargo" constraint. Each

row will have three nonzero coefficients that represent possible transportation via airlift,

truck, and/or rail. The total amount of cargo (in ton-miles) that must be moved on a given

day t is represented by the right-hand side value of b_{tc}.   For example, $b_{1c} = 741{,}255$

would indicate that on *day one* of the deployment, 741,255 ton-miles of cargo must be

delivered within the theater.   The symbol "$a_{t(1c)}$" represents an A matrix coefficient and is

the capacity of a C-130 assigned to carry cargo on day t. The subscript t indicates which

day's requirements are being moved and the subscript (1c) indicates which type of vehicle

is moving the cargo. For example, $a_{1(1c)} = 13{,}500$ indicates that the capacity of vehicle

type one (C-130 airlift) on day one is 13,500 ton-miles. Likewise, $a_{1(2c)} = 3{,}240$ indicates

that the capacity of vehicle type two (trucks) on day one is 3,240 ton-miles.   The

coefficients and right-hand-side are loaded into the formulation from an input cell as shown in Figure IV-1.

Constraint number (2) in Figure III-1 ensures that the passenger requirement is met. This means that enough vehicles must be assigned with sufficient capacity to move the required passengers for a given day using the following formulation:

$$a_{t(1p)}(X1P\_t) + a_{t(2p)}(X2P\_t) + a_{t(3p)}(X3P\_t) \geq b_{tp}; \; t = 1,. . ., n$$

This constraint set also consists of n rows. A single row from this formulation is called a "day t passenger" constraint. The A matrix coefficients are similarly expressed, except that the number now represents a vehicle's capacity to move passengers. Likewise, the right-hand side in constraint (2), b_{tp} is now the number of passengers that must be moved on day t.

Constraint number (3) in Figure III-1 ensures that the aeromedical patient requirement is met. This means that enough vehicles must be assigned with sufficient capacity to move the expected aeromedical patient requirement for a given. The constraint's formulation is:

$$a_{t(1m)}(X1M\_t) + a_{t(2m)}(X2M\_t) + a_{t(3m)}(X3M\_t) \geq b_{tm}; \; t = 1,. . ., n$$

This formulation also forms n rows in the A matrix and the coefficients now represent a vehicle's capacity to move casualties and the right-hand-side is the expected number of casualties for a given day that must be moved.

Constraint number (4) uses the Xp variable as a means to find the maximum number of C-130s used at any given time for all of the periods involved in the model's

planning horizon, which is set by "n". Thus, constraint (4) will form n total constraints, one for each day:

$$X1C\_t + X1P\_t + X1M\_t \leq Xp; \quad t = 1,\ldots, n$$

As an example, if the variable $Xp = 50$, then the greatest number of C-130s that are scheduled for any day in the period of n days is fifty. So, for each day in the scenario, fifty or less C-130s were used.

Constraints (5) and (6) are identical in concept and formulation to constraint (4), except that constraint (5) uses the variable set associated with trucks and constraint (6) uses the variable set associated with trains, yielding the maximum number of trucks and trains used, respectively.

Constraint number (7) is an optional constraint that can be employed to place a maximum or upper limit on the number of C-130s used on a given day. This constraint is written: $Xp \leq b_{UL\ C-130}$. For example, if a maximum of 145 C-130s could be used for a contingency, then set $b_{UP\ C-130} = 145$ and the constraint is: $Xp \leq 145$. Constraint numbers (8) and (9) are identical in concept and differ in that (8) places an upper limit on trucks and (9) places an upper limit on 22-car trains.

Constraint number (10) is an optional constraint that can be employed to place a minimum or lower limit on the number of C-130s used on a given day. This constraint is written: $Xp \geq b_{LL\ C-130}$. For example, if one wanted to see the effect of employing at least fifty C-130s on at least one day of a contingency, then set $b_{LL\ C-130} = 50$ and the constraint is: $Xp \geq 50$. Constraint numbers (11) and (12) are identical in concept and

differ in that (11) places a lower limit on trucks and (12) places a lower limit on 22-car trains.

Constraint (13) is used to force a percentage of the transportation requirements to be allocated to C-130 airlift transporting cargo and is called a "must air" constraint for cargo:

$$\sum_{t=1}^{n} a_{t(1c)}(X1C\_t) \geq \alpha_c \sum_{t=1}^{n} b_{tc} ; \text{ where } 0 \leq \alpha_c \leq 1$$

A similar formulation can be used for passengers and medical evacuation patients as shown by constraints (14) and (15). The right-hand-sides multiplies a desired fraction with the total sum of daily requirements for either cargo, passengers, or medical evacuation patients. The user sets the fraction α depending upon what portion of the requirement must be carried via C-130. The left hand side sums the number of C-130s for a particular day times the capacity for the C-130. If the constraint is being used for cargo, then the capacity $a_{t(1c)}$ is the cargo capacity for a C-130. If the constraint is being used for passengers, then the passenger capacity is entered as the coefficient $a_{t(1c)}$.

Constraint (16) is used to specify an upper limit for an operating budget for C-130 aircraft. Since this is an Air Force sponsored thesis, it is assumed the user of this formulation is not interested in the operating budgets of the other modes of transportation. This constraint is formulated as follows:

$$(\text{Ute rate})(\text{hourly operating cost}) \sum_{t-1}^{n} (X1C\_t + X1P\_t + X1M\_t) \leq (\text{Budget})$$

A utilization rate (Ute rate) is entered as the number of hours a C-130 operates per day. An hourly operating cost is entered as the number of dollars per hour. This allows the left

hand side to represent the operating cost of all the C-130s being used by a given solution, once the variables X1C_t, X1P_t, and X1M_t have been determined. A budget is entered as the right-hand-side and represents the allowable budget for a contingency of *n* days that the user must operate under. As a sidenote, the value used for a peacetime planning utilization rate for C-130 aircraft is 10 hours and the hourly cost of this vehicle is \$2,215/hour for Department of Defense uses (USAF AMS, 1992:204-6).

Minimize $z = w_p Xp + w_t Xt + w_r Xr$

Subject To

$a_{t(1c)}(X1C\_t) + a_{t(2c)}(X2C\_t) + a_{t(3c)}(X3C\_t) \geq b_{tc}$; $t = 1,\ldots, n;$ Cargo constraints (1)

$a_{t(1p)}(X1P\_t) + a_{t(2p)}(X2P\_t) + a_{t(3p)}(X3P\_t) \geq b_{tp}$; $t = 1,\ldots, n;$ Passenger constraints (2)

$a_{t(1m)}(X1M\_t) + a_{t(2m)}(X2M\_t) + a_{t(3m)}(X3M\_t) \geq b_{tm}$; $t = 1,\ldots, n;$ Casualty constraints (3)

$X1C\_t + X1P\_t + X1M\_t \leq Xp;$ $t = 1,\ldots, n;$ Maximum C-130s constraints (4)

$X2C\_t + X2P\_t + X2M\_t \leq Xt;$ $t = 1,\ldots,n;$ Maximum trucks constraints (5)

$X3C\_t + X3P\_t + X3M\_t \leq Xr;$ $t = 1,\ldots, n;$ Maximum rail constraints (6)

$Xp \leq b_{\text{UL C-130}}$ C-130 upper limit constraint (7)

$Xt \leq b_{\text{UL Trucks}}$ Truck upper limit constraint (8)

$Xr \leq b_{\text{UL Rail}}$ Rail upper limit constraint (9)

$Xp \geq b_{\text{LL C-130}}$ C-130 lower limit constraint (10)

$Xt \geq b_{\text{LL Trucks}}$ Truck lower limit constraint (11)

$Xr \geq b_{\text{LL Rail}}$ Rail lower limit constraint (12)

$\sum_{t=1}^{n} a_{t(1c)}(X1C\_t) \geq \alpha_c \sum_{t=1}^{n} b_{tc}$; where $0 \leq \alpha_c \leq 1$; C-130 "must air" cargo constraint (13)

$$\sum_{t=1}^{n} a_{t(1p)}(X1P\_t) \geq \alpha_p \sum_{t=1}^{n} b_{tp} \; ; \quad 0 \leq \alpha_p \leq 1; \; \text{C-130 "must air" passenger constraint} \quad (14)$$

$$\sum_{t=1}^{n} a_{t(1m)}(X1M\_t) \geq \alpha_m \sum_{t=1}^{n} b_{tm} \; ; \quad 0 \leq \alpha_m \leq 1; \; \text{C-130 "must air" casualty constraint} \quad (15)$$

$$\text{(Ute rate)(hourly operating cost)} \sum_{t-1}^{n} (X1C\_t + X1P\_t + X1M\_t) \leq \text{(Budget)} \quad (16)$$

{X1C_t, X1P_t, X1M_t, X2C_t, X2P_t, X2M_t, X3C_t, X3P_t, X3M_t,

Xp, Xt, Xr} $\in \mathfrak{R}_{+}$

Figure III-1 Spreadsheet Formulation

There are 9(n) + 3 variables and a total of 6(n) + 10 possible constraints in the formulation. The limit constraints; (7), (8), (9), (10), (11), and (12), can be omitted if the user wishes to know the unconstrained minimum number of vehicles required to move a given amount of cargo, passengers, and aeromedical patients.   If the user wants to put a numerical limit on one or more of the vehicles, then the appropriate limit constraints would be employed in the formulation. For example, if the user wanted to limit the C-130s to between 50 and 150, he or she would include constraints (7) and (10). The user would set $b_{\text{UL C-130}} = 150$ in constraint (7) and $b_{\text{LL C-130}} = 50$ in constraint (10). Various combinations could be employed depending on the scenario desired by the user.

Likewise, the "Must Air" constraints and the Budget constraint can also be omitted if they are not pertinant to the analysis at hand.

The nature of the problem obviously requires an integer answer. Excel requires

an excessive amount of time to solve formulations with a large number of variables constrained to integer values. Consequently, for a rough cut look, the integer requirement has been relaxed. The EXCEL© solver is set up to use nonnegative real numbers as the domain for the variables. The solver dialogue box takes approximately one to two minutes to display and the problem takes approximately five minutes to solve. The problem could be recast into an integer format that is acceptable for input into a more powerful solver such as CPLEX©. The example case has been cast in MPS format for this purpose. This example integer formulation was solved using CPLEX© on a SPARC 10 workstation in 282.25 seconds using 56,213 iterations and 20,000 nodes. MPS format is a long established format for mainframe linear programming systems and is therefore a convenient format to use for this problem. (CPLEX©, 1994:81) This format can be used for many other solvers besides CPLEX©, although CPLEX© uses at extended version of the standard MPS format which may not be accepted by older linear programming codes. The extended version formulated for CPLEX© was recast into a basic MPS format which is accepted by all linear programming codes.

IV. Data Description and Analysis

IV.1. Excel Spreadsheet Formulation

The main motivation for the spreadsheet formulation was to allow the user to input the required data in a worksheet that was as self-explanatory as possible. The worksheet would link the cells containing the input data to a worksheet used for solving the problem. This worksheet would be formatted in a way that was suitable for input into the spreadsheet solver. The next page shows an example of the input fields for the spreadsheet formulation:

Table IV-1 Spreadsheet Model Input Page

i	Type	Tons or #	Ton-miles				
				Thesis Model: C130/Rail/Truck			
				Korean Scenario			soldier:
	Operating Costs			C-130 data			Avg.wt(#)
		Total	Per hour	C-130 ute rate:	10	Upper	200
	C-130	4.00E+06	2215	speed:	270	Limit	
				productivity factor	0.40	1000	
				cargo capacity (tons)	13	Lower	
Pusan-DMZ:		210	avg. dist.	105	ton-mile capacity:	13500 Limit	
	TPFDD Data			Cycles (trips) / day	2	0	
i	Type	Tons or #	Ton-miles	# pax per chalk	90		
day 1	Cargo:	7060	741255	**pax capacity per day**	180		
	Pax:	4794		pax cap. ton-miles	19440		
	Medical:	1240		# patients per chalk	70		
day 2	Cargo:	8191	860005	**Aeromed. cap./ day**	140		
	Pax:	5782		Aeromed. cap. (tons)	15120		
	Medical:	1409		% cargo that must go by C-130		0	
day 3	Cargo:	8177	858621	% passenger "must air"		0	
	Pax:	3838		% medevac "must air"		0	
	Medical:	1266					
day 4	Cargo:	8005	840552	**Truck data**		Upper	
	Pax:	4745		Trips per day:	2	Limit	
	Medical:	1389		capacity (tons):	20	1000	
day 5	Cargo:	7386	775491	miles per trip:	90	Lower	
	Pax:	4164		Availability:	0.9	Limit	
	Medical:	1527		**ton-mile capacity:**	3240	0	
day 6	Cargo:	8762	920041	# pax per truck	45		
	Pax:	4676		**daily pax capacity**	90		
	Medical:	1464		# patients per truck	7.5		
day 7	Cargo:	6072	637554	**Medevac capacity**	15		
	Pax:	4287					
	Medical:	1607					
day 8	Cargo:	6562	688998	**Rail data**		Upper	
	Pax:	4936		(1 train = 22 flatcars)		Limit	
	Medical:	1487		Trips per day:	2	1000	
day 9	Cargo:	5838	613014	capacity (tons)	1210	Lower	
	Pax:	5290		miles per trip:	100	Limit	
	Medical:	1472		ute rate	0.75	0	
day 10	Cargo:	8060	846263	**ton-mile capacity**	181500		
	Pax:	4539		(1 train = 22 passcar)			
	Medical:	1637		Trips per day:	2		
day 11	Cargo:	7852	824506	pax capacity (count)	413		
	Pax:	5210		**daily pax capacity**	826		
	Medical:	1443		**daily medevac cap.**	200		
day 12	Cargo:	5796	608594	(1 train = 22 gondolas)			
	Pax:	5220		Trips per day:	2		
	Medical:	1491		capacity (tons)	1100		
day 13	Cargo:	7154	751196	miles per trip	100		
	Pax:	3672		ute rate	0.75		
	Medical:	1622		ton-mile capacity	165000		
day 14	Cargo:	6168	647684	(1 train = 22 boxcars)			
	Pax:	4553		Trips per day:	2		
	Medical:	1605		capacity (tons)	1100		
				miles per trip	100		
				ute rate	1		
				ton-mile capacity	220000		

The spreadsheet input page allows the user to build the linear program without having to keep track of the model structure. The appropriate cells from Table IV-1 are

linked in the spreadsheet to the appropriate cell in the model formulation. The cells of the model formulation are actually used to solve the problem and have been entered into the EXCEL spreadsheet solver. It should be noted that Table IV-1 only shows *fourteen* days of input, while twenty days are used for the results provided in this chapter.

The input data listed in Table IV-1 is notional. The requirements for this notional scenario have been set to about 4,600 personnel per day, a cargo flow of approximately 7,000 tons per day, and a total casualty rate of around 1,400 per day. The input data was created using random draws from a normal probability distribution. Assuming a Korean scenario would require a military effort commensurate to the Gulf War of 1990, the average daily personnel requirement was used since approximately this number of personnel were moved by intratheater airlift in the Gulf War of 1990. The casualty rate is from the author's memory of predictions made prior to the Gulf War of 1990 and from the casualties one could expect if currently deployed forces in South Korea could not suppress the initial North Korean attack. The intratheater cargo requirement comes from a study of Desert Storm intratheater cargo lift which averaged about 7,000 tons per day (USAF AMS, 1992:430-1). Daily cargo requirements were randomly drawn from a normal probability distribution with a mean of 7,000 and a standard deviation of 15%. Daily personnel requirements were randomly drawn from a normal probability distribution with a mean of 4,644 and a standard deviation of 15%. The first two weeks of daily casualties were drawn from a uniform probability distribution with a low value of 1,215 and a high value of 1,643. The remaining casualty figures for days 15 to 20 came from a uniform probability distribution with a low value of 275 and a high value of 371. The vehicle

capacities are reasonable numbers. The data under the cell labeled "C-130 Data" comes

from the Air Mobility Command's USAF Air Mobility School Learning Guide and Air

Force Pamphlet 76-2 "Airlift Planning Factors."   The capacity

of a C-130 is figured using an aircraft utilization rate, aircraft planning speed, a

productivity factor, and a cargo capacity accepted as a planning figure. These factors are

used for a "ton-mile" capacity based on the common use of this performance measure in

airlift capability calculations (USAF AMS, 1992:403-1). Straight tons could also be used;

however, a ton-mile calculation allows utilization rates and productivity factors to be

incorporated into the problem.   A ton-mile figure also implicitly incorporates the size of

the theater by using an "average travel distance of lift" as a factor with the amount of

cargo to be moved. A way to see this idea is to imagine one ton of cargo requiring

transportation. If the cargo must be moved 200 miles, then 200 ton-miles are needed. If

the cargo must be moved 400 miles, then 400 ton-miles are needed. As can be seen, the

range of an airlift aircraft can also be implicity represented by a capacity expressed in

ton-miles instead of just weight alone.

The utilization (UTE) rate is the total hours of capability per aircraft a fleet of

airlift aircraft can produce in a day expressed in terms of Primary Authorized Aircraft

(PAA). For example, if we expect to need 4,000 hours per day of flying time with the C-

130 fleet and we have a fleet of 400 C-130s (PAA = 400), then the UTE would be 10

hours. A UTE rate of 10 hours is normally used for notional planning, and it should be

noted that actual UTE rates are classified. A productivity factor is based on historical data

and measures what part of the UTE rate is actually spent with a load in the back of the

aircraft (USAF AMS, 1992:403-2). In Table IV-1, we use a number of 0.4 to indicate that 40% of the UTE rate occurs with a load and the other 60% is an aircraft flying empty. These numbers happen to be the recommended percentages used for peacetime planning of intratheater airlift (USAF AMS, 1992:403-2). This would account for positioning and depositioning legs. Normally, the onload and offload bases are not the locations an aircraft launches from or recovers to at the end of a mission. Before entering numbers into these capacity cells, the analyst would be well advised to make a thorough study of Air Force Pamphlet 76-2, because use of maximum capability numbers could yield an overly optimistic result. Note that wartime UTE rates must be determined from classified sources and entered here in order to conduct a proper wartime analysis. The model uses the figure from the cell labeled "ton-mile capacity" which incorporates the factors listed above it. The passenger and aeromedical/medical capacities are just the upper limit of passengers or patients that would be carried in one C-130 leg multiplied by the expected number of such trips the planner or analyst expects the fleet to make on average.

The truck capacities in Table IV-1 come from an unclassified briefing called SUMMITS obtained from Major John Stievens at Air Combat Command on 12 October 1995.

The rail capacities come from simulation studies conducted by Dr Elizabeth N. Abbe of the US Army Concepts Analysis Agency, Bethesda, MD for a mobility conference held at the Air Force Institute of Technology in May, 1995. From a letter dated 14 November 1995, Dr Abbe stated that a published report of the mobility study is

forthcoming in a few months. The simulation is called GDAS or the Global Deployment Analysis System, which is a high resolution, multi-modal entity model for the comprehensive simulation of end-to-end force deployment. The capacities used for rail in this study were for 22-car train entities where all the cars of an entity were of the same type. (Abbe, 1995:1) This study adopted the same convention for convenience. The user may want to modify this approach to modeling rail capacities.

The right-hand-sides for the linear program are linked to Table IV-1 in the columns adjacent to the column with the "Day t" labels. These numbers would be entered by the user from a breakout of the TPFDD or based on expected sustainment rates required for the contingency under study. Obviously, the most suspect numbers in the table are the cells labeled "Medical." The analyst would have to consider all the classical factors of warfare and the expectations of the leadership before arriving at these casualty figures.

The following is a chart of the daily cargo requirements as broken out from the spreadsheet input page (This data is notional as previously described and is not classified):
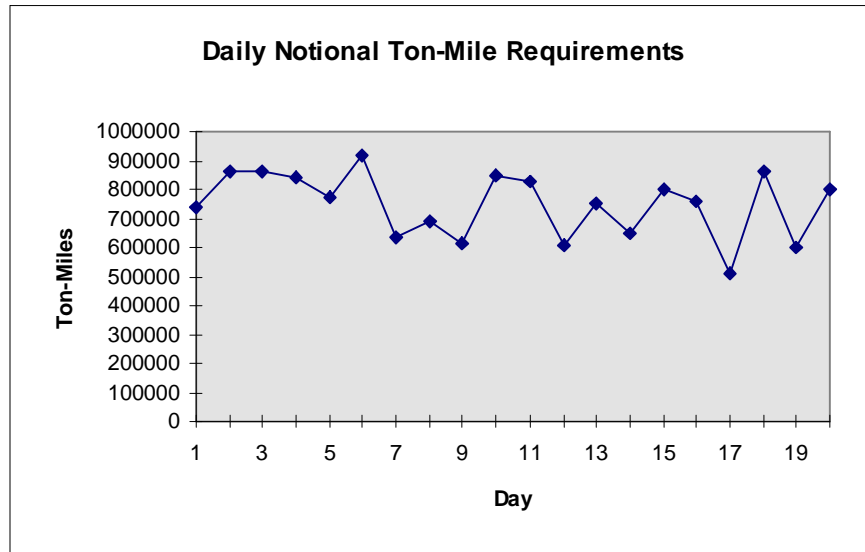
Figure IV-1 Daily Cargo Requirements (Notional Data)

Below is a chart of the daily passenger requirements as broken out from the spreadsheet input page. (This data is notional as previously described and is not classified):
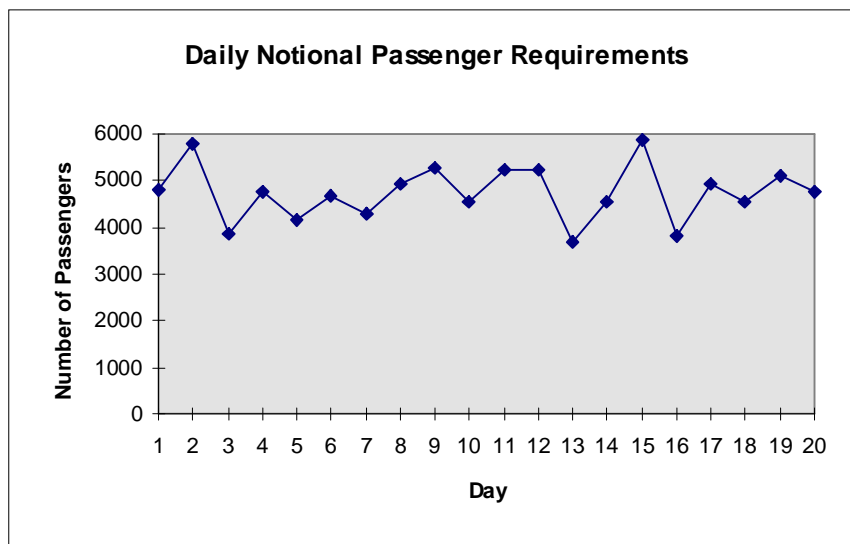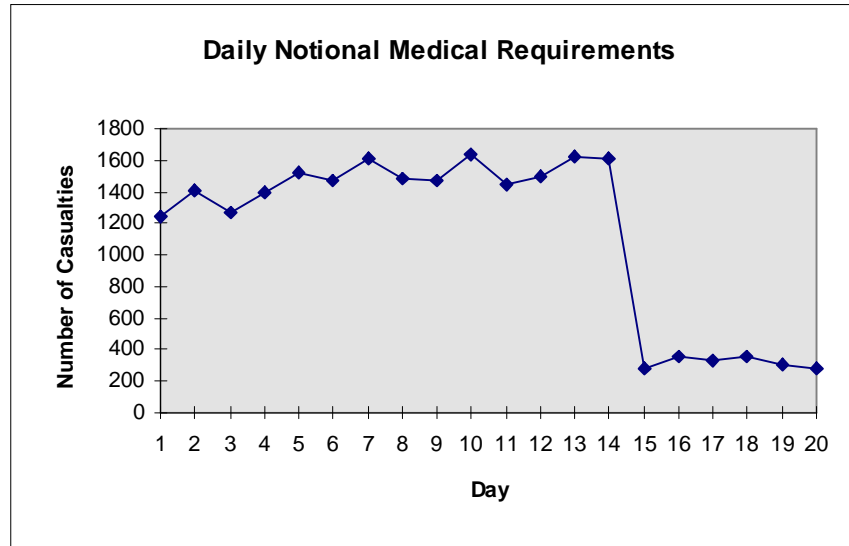

Figure IV-2 Daily Passenger Requirements (Notional Data)

Figure IV-3 is a chart of the daily medical evacuation requirements as broken out from the spreadsheet input page. (This data is notional as described previously and is not classified):

Figure IV-3 Daily Medical Evacuation Requirements (Notional Data)

MPS input files were needed to enter the spreadsheet model data into the CPLEX© and LINDO© solvers. CPLEX© was used to verify the solver used in EXCEL©. LINDO© was used to conduct the parametric analysis in the following sections of this chapter.

Another data file of interest was created to see the effects of a peak in demand for cargo. Figure IV-4 shows a graph of this particular data. Note that there is a surge of demand occuring in week one. The peak demand was also taken from random draws on a normal probability distribution with higher means used to affect the surge.
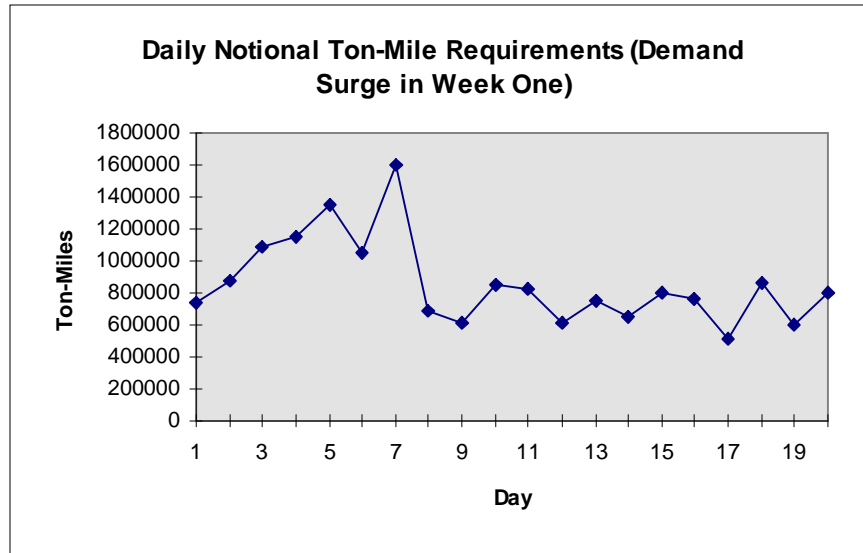
Figure IV-4 Daily Cargo Requirements (Notional Data) - Surge Problem

IV.2. *Spreadsheet Parametric Analysis*

The LINDO© solver was the only application available to the author that could accomplish a parametric analysis on a linear program.   A parametric analysis was accomplished for both the objective function coefficients and the right-hand-side values. First, the method used for conducting a parametric analysis on the objective function coefficients is addressed.

LINDO© essentially performs a parametric analysis on objective function coefficients by optimizing different objective functions as specified within the model. This is analogous to choosing different objective functions of interest and running an individual optimization for each objective function. By representing the objective function with a single variable, an objective function can be expressed as a constraint. Then the model is solved using a single variable in the objective function that is also found within the

constraint. Below is an example of how to formulate the objective function Minimize $Z =$ $Xp + 2Xt + 5Xr$ with a single variable in the objective function: (LINDO©, 1991:44)

MIN Z

SUBJECT TO

Z - Xp - 2Xt - 5Xr = 0

In order to include numerous objective functions, one would specify all the desired objective functions in the form of constraints and change the variable in the objective function appropriately before solving the problem.   For example, one could add the following objective functions to the model above:

(1) Minimize Z = 2Xp + Xt + 11Xr

(2) Minimize Z = 4Xp + Xt + 15Xr

(3) Minimize Z = Xp + 10Xt + 20Xr

The functions (1), (2), and (3) will be represented by the variables Y, X, and W respectively and modeled as follows:

MIN Z

SUBJECT TO

Z - Xp - 2Xt - 5Xr = 0

Y - 2Xp - Xt - 11Xr = 0

X - 4Xp - Xt - 15Xr = 0

W - Xp - 10Xt - 20Xr = 0

If the objective function (1) needed to be solved, the expression "MIN Z" would be changed to read "MIN Y". If the objective function (2) were to be used, the expression would then become "MIN X". (LINDO©, 1991:44-45)

Before candidate objective functions are explicitly written into the model as stated above, a preliminary parametric analysis on objective function coefficients was accomplished by varying the coefficients in the EXCEL© spreadsheet model. The results are shown in Table IV-2 below.

Table IV-2 Parametric Analysis of Objective Function

Coefficients of Objective Function Variable:			Max Number of vehicles on any given day for:		
Xp	Xt	Xr	C-130	Trucks	Trains
1	1	1	0	0	18.78
1	1	5	42.19	0	4.75
1	1	10	42.19	0	4.74
1	1	15	105.89	0	0
1	1	20	105.89	0	0
5	1	5	0	0	18.78
5	1	10	10.06	62.30	4.95
5	1	15	10.06	62.30	4.95
5	1	20	10.06	62.30	4.95
5	1	25	10.06	62.30	4.95
5	1	30	10.46	62.07	4.89
5	1	35	10.46	62.07	4.89
5	1	40	10.46	62.07	4.89
5	1	45	10.46	62.07	4.89
5	1	50	10.46	62.07	4.89
5	1	55	10.46	62.07	4.89
5	1	60	10.46	335.91	0

As can be seen from Table IV-2, the solution is sensitive to the choice of objective function coefficients. If all the decision variables are equally weighted, as in the first line

of Table IV-2, then the model allocates everything to 22-car trains for this notional example. This makes sense when one considers that 22-car trains have the largest capacity per vehicle and the objective is to minimize the number of vehicles used. It has also been the author's experience in working with the US Army that they would prefer to move on rail vehicles with other factors being equal. Unfortunately, other factors may become paramount to the decision maker which makes the use of rail more costly or "expensive." This situation is represented by differing the weights of the decision variables in the objective function. Note that when the objective coefficient for rail becomes sufficiently large, everything is allocated to C-130s. A look at some possible factors that must be captured by the objective function coefficients will illustrate the greatest challenge of the spreadsheet model.

Some factors that could impact the weight of the decision variables are: 1) locations of rail terminals v. US Army assembly areas v. airlift terminals; 2) Speed of the respective transportation; 3) the criticality of transportation requirements during various portions of the planning horizon; 4) Vulnerability of transportation routes to enemy interdiction operations with respect to each mode of travel; 5) tactical considerations that cause a decision maker to prefer one mode of travel over another; and 6) the flexibility of a mode of travel, i.e., how easy is it to reroute a vehicle to a new destination. This thesis does not thoroughly investigate a systematic method for determining an appropriate method for deciding the weights of the objective function for this model. This would be an area for future research.

To conduct a parametric analysis of the right-hand-sides, LINDO© provides a command called "PARA" that invokes a routine which varies a selected row right-hand-side from the current value to a value input by the user. The output of this routine shows the change in optimal objective value for each basis generated. Each basis is generated within the range of right-hand-side values selected and the row selected after executing the PARA command in LINDO©. Each change in objective function value is accompanied by a change in basis. In this manner, the user can automatically trace out the effect of varying the right-hand-side over a wide range (LINDO©, 1991:42-44).

Since the application envisioned in this thesis implies varying any number of right-hand-side values simultaneously, the analysis in this section made use of a device called a D-vector before using the PARA command in LINDO©. A constraint was written into the model as $D = 0$. This would constitute the row whose right-hand-side would be varied with the PARA command and adds an extra variable, D, to the problem. In addition, a D-vector in the form of an added column to the problem was constructed. (LINDO©, 1991:44) It is important to note that the values entered in this column as coefficients of the variable D are chosen arbitrarily. Their values are determined depending upon what changes in right-hand-side values the user desires to study.

IV.3.   Analysis of a Potential Scenario for Parametric Analysis

This analysis assumed the decision maker had chosen the objective function weights represented by the equation, Minimize $Z = 5X_p + X_t + 30X_r$. As can be seen from the weights of this equation, rail transport was six times as expensive an asset as C-

130 airlift and thirty times as expensive as truck transport. In turn, the C-130 mode was five times as expensive as the truck mode.

Before the parametric analysis of the right-hand-sides is presented, solutions of various realizations are presented. The first set of solutions shows the number of vehicles assigned when the original data set (Figures IV-1, IV-2, and IV-3) is used. The next set of solutions shows the number of vehicles assigned when the surge cargo data set in Figure IV-4 is used in place of the data in Figure IV-1. This set of solutions is labeled the "Surge Problem." The third set of solutions shows the number of vehicles assigned when constraint (13) is introduced such that all cargo must be moved by airlift vehciles (see Figure III-1). This set of solutions has the label "Must Air" for cargo. The fourth set of solutions shows the number of vehicles assigned when the budget constraint (16) is employed to restrict operating costs to $4 million.
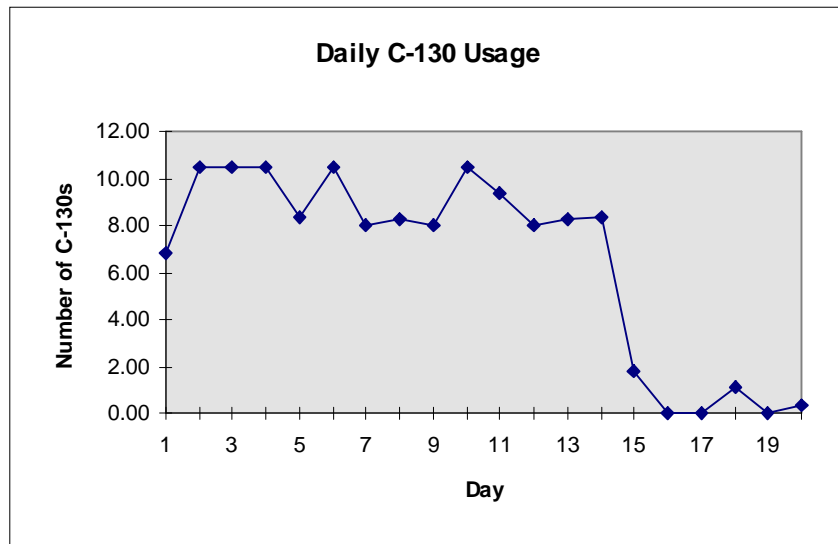
Figure IV-5 Daily C-130s Used - Original Problem

In Figure IV-5, the maximum number of C-130s used on any given day is 10.46, i.e., $X_p = 10.46$. Note that this value is reached on days 2, 3, 4, 6, and 10 only and that the number of vehicles assigned decreases dramatically after day 14. To understand this solution better, it is instructive to examine a graph where the type of loads carried are broken out as in Figure IV-6. Notice that most of the C-130s carry casualties. From Figure IV-3, it is seen that the medical evacuation requirements also drop off dramatically after day 14.
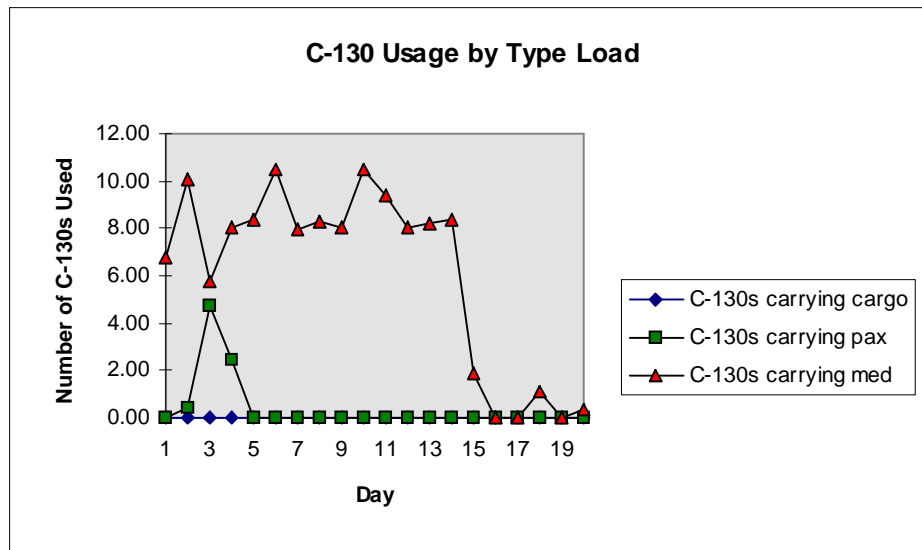
Figure IV-6 C-130 Usage by Type Load
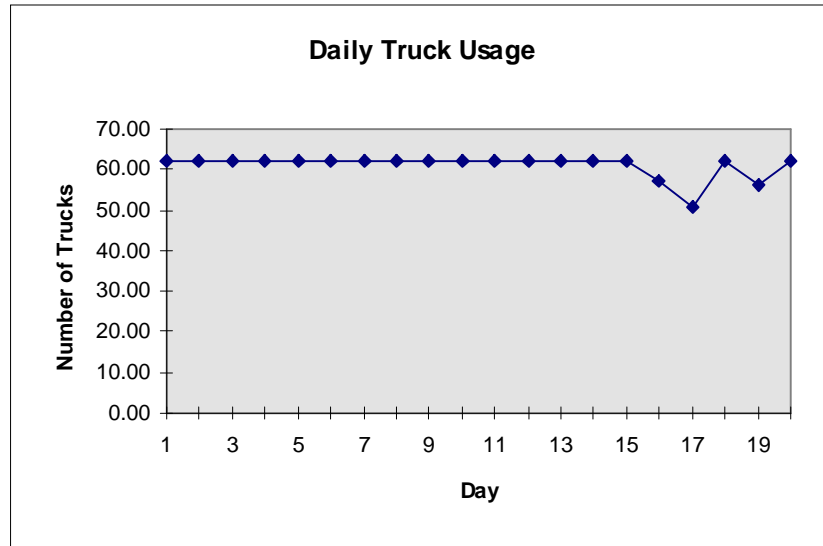
Figure IV-7 Daily Trucks Used - Original Problem

The maximum number of trucks assigned on any given day is 62.07, i.e., $Xt = 62.07$. As shown from Figure IV-8, the majority of trucks are assigned to carry passengers and only on one day are trucks assigned to carry cargo. Also note that where trucks carry casualties, less trucks are assigned to carry passengers.
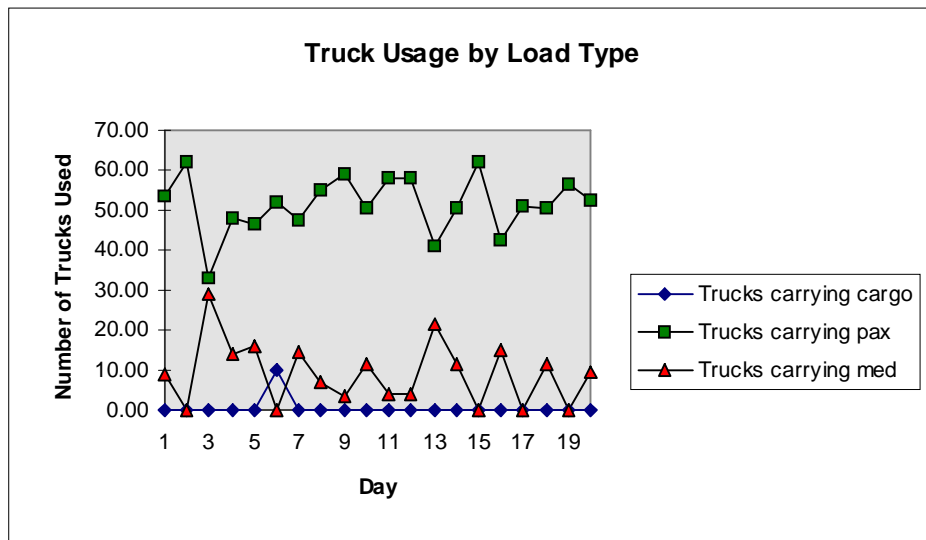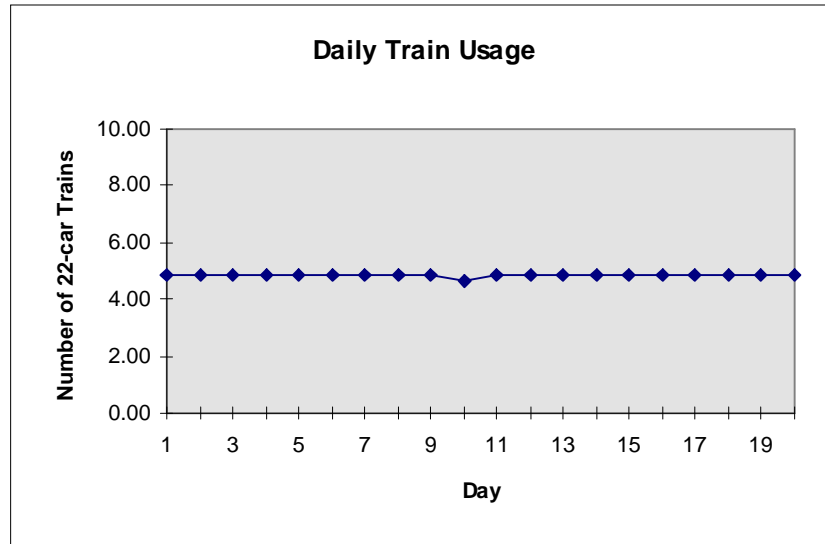

Figure IV-8 Truck Usage by Load Type

Figure IV-9 Daily 22-Car Trains Used - Original Problem

The maximum number of 22-car trains used for any given day is 4.89, i.e., X_r = 4.89. In Figure IV-10, it is seen that most of the 22-car trains are assigned to carry cargo with only a small fraction carrying passengers on days 15, 17, and 19. Close to one 22-car train is assigned to medical evacuation on eight days, but again, the major operation for trains in this scenario is cargo transportation.
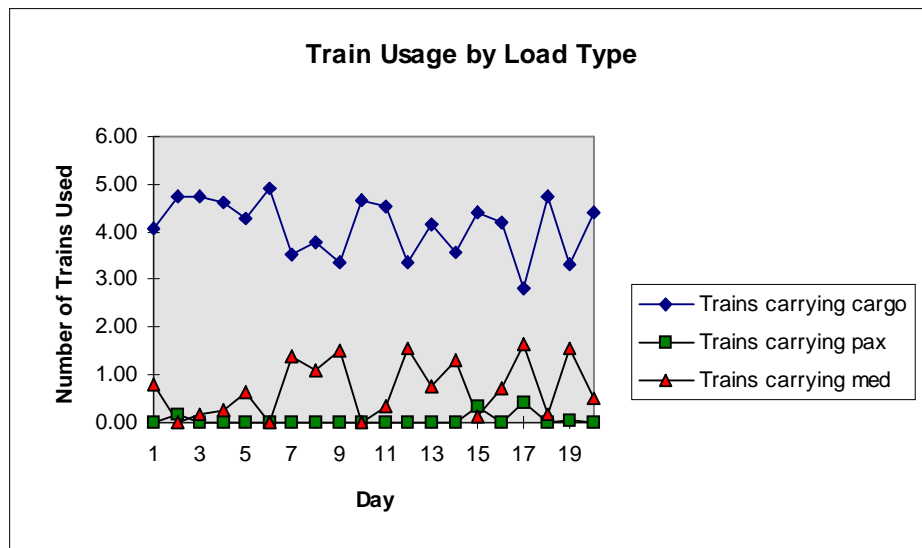

Figure IV-10 Train Usage by Load Type

If the capacities and objective function weights are examined, it becomes apparent that this scenario will always give the cargo to trains and medical evacuation primarily to C-130s, while the trucks generally get assigned to carry passengers for this scenario's data set.

Even though the trains are much more heavily weighted than the other modes of transportation, and therefore more expensive, their relative capacity for cargo is so much greater than the other vehicles that the objective function is minimized by assigning cargo mainly to trains. However, in the case of passengers and medical evacuation transportation, the capacity of trains is not large enough relative to trucks and C-130s to overcome the weight assigned trains. Therefore, the objective function is minimized when vehicles other than trains are used for passengers and casualties. The same type of relative interaction occurs between trucks and C-130s when the model assigns vehicles to carry passengers and casualties. Here it is seen that C-130s are five times as costly to assign as trucks, so the capacity of a C-130 would have to be relatively large relative to a truck in order to get assigned to provide transportation. The C-130s get assigned many medical evacuation missions because it has over nine times more medical evacuation capacity than a truck. When it comes to passengers, however, the C-130 has only twice the capacity of a truck and this cannot overcome the weighting of the objective function.

The solution sets that follow this paragraph are not broken out by types of loads; however, they follow the same general pattern as described above. The primary difference is due to the governing constraint introduced or the change in input data used. They are listed to demonstrate effects of varying constraints or input data. Although not shown in

this thesis, it must be pointed out that all these solution sets have multiple optimal solutions when it comes to the portion of the planning horizon's solution where maximum vehicle assignments are *not* made. Each solution to a problem will have the exact same value for Xp, Xt, and Xr, but they may differ as to the number of vehicles assigned on days where this maximum assignment is not made.

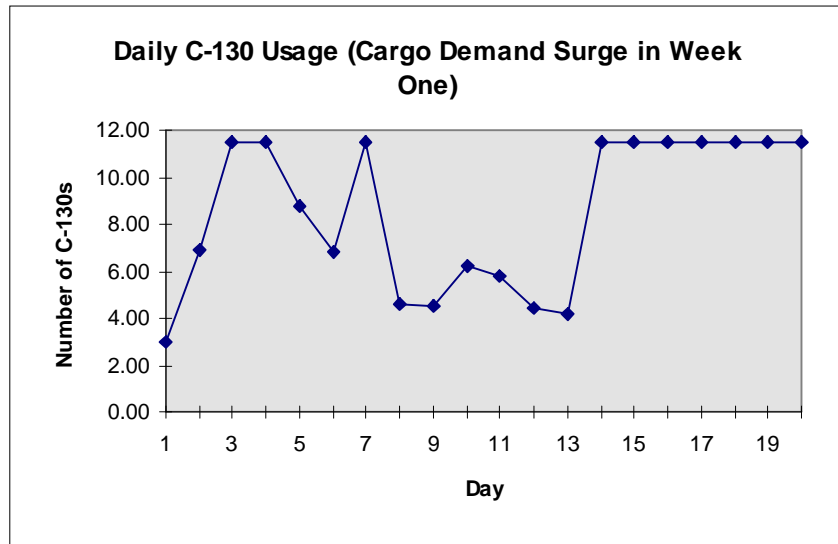SOLUTION USING DATA FOR CARGO DEMAND SURGE IN WEEK ONE:

Figure IV-11 Daily C-130s Used - Surge Problem

Here, $X_p = 11.48$. This is a little larger than the value in the original problem.
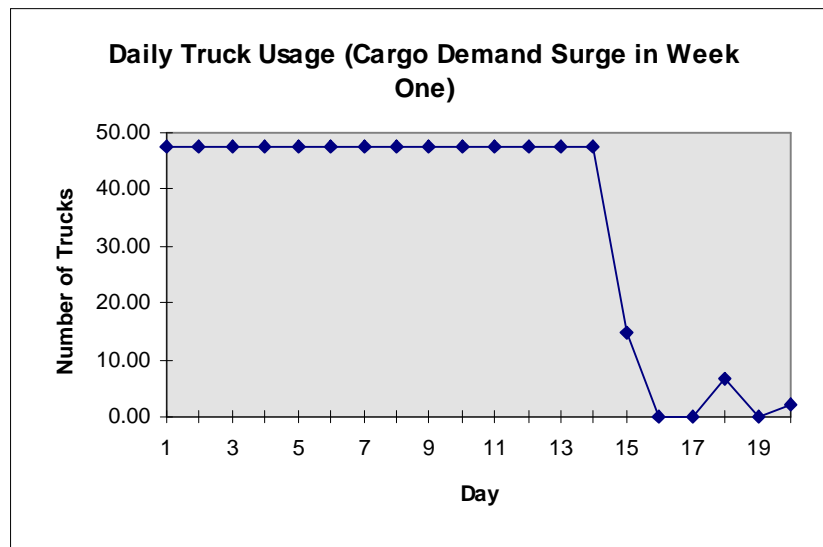

Figure IV-12 Daily Trucks Used - Surge Problem

$X_t = 47.64$, a smaller maximum on trucks than in the original problem.
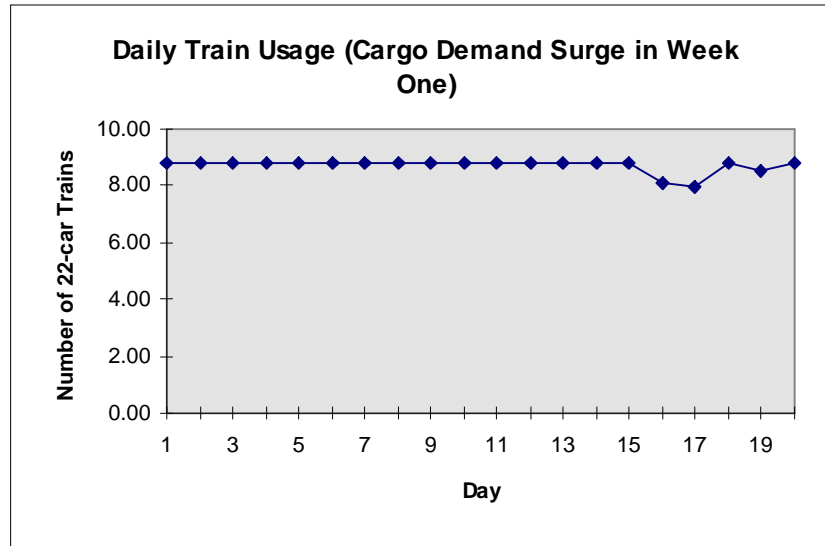
Figure IV-13 Daily Trains Used - Surge Problem

$Xr = 8.82$, which is considerably larger than the original problem when the relative capacity is considered. It is not surprising that the trains increased the most with an added requirement for cargo incorporated into the problem. As noted before in the original problem, with the weights of the objective function we are using, there is a propensity for the model to assign virtually all the cargo to trains. Therefore, an increase in cargo should create an increase in trains assigned.
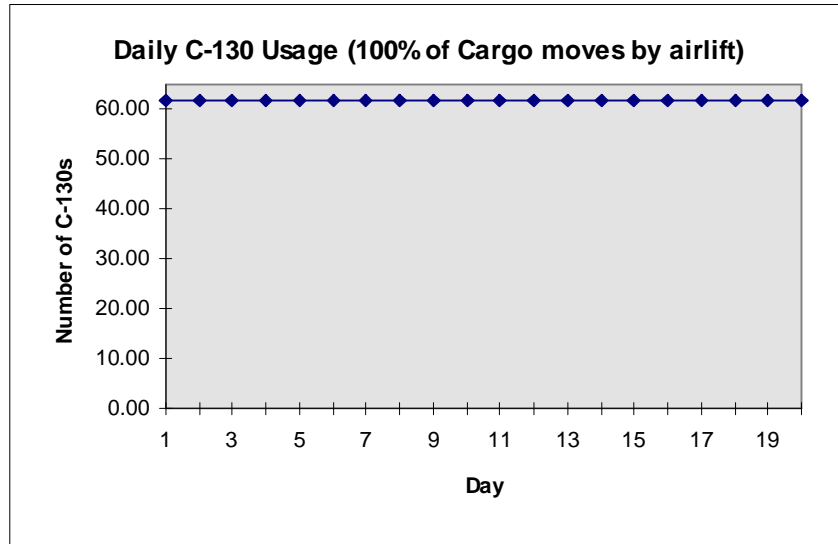
SOLUTION USING 100% MUST AIR CONSTRAINT FOR CARGO:

Daily C-130 Usage (100% of Cargo moves by airlift)

Figure IV-14 Daily C-130s Used - "Must Air" for Cargo

$Xp = 61.94$ in Figure IV-14. As can be seen above, the "Must Air" constraint for cargo dramatically increases the number of C-130s assigned. In this case, the objective function weights give most of the medical evacuation missions to C-130s, but the introduction of the "Must Air" constraint also gives virtually all of the cargo missions to C-130s. Had the constraint been formulated with an individual constraint for each day, all the cargo would go to C-130s, so the surplus that allowed trains to get a portion of the cargo was due to formulating all variables into one constraint, i.e., one constraint covers all the periods modeled in the problem. While enough C-130s are assigned to carry the weight of the cargo demand, some of the daily cargo constraints still had a surplus, i.e., they were nonbinding. The trains picked up the surplus on days that C-130s did not carry all cargo.
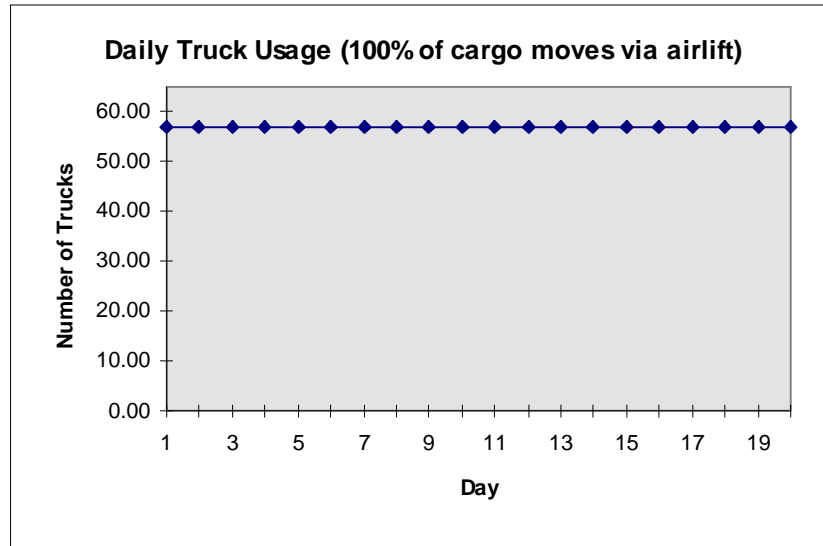
Figure IV-15 Daily Trucks Used - "Must Air" for Cargo
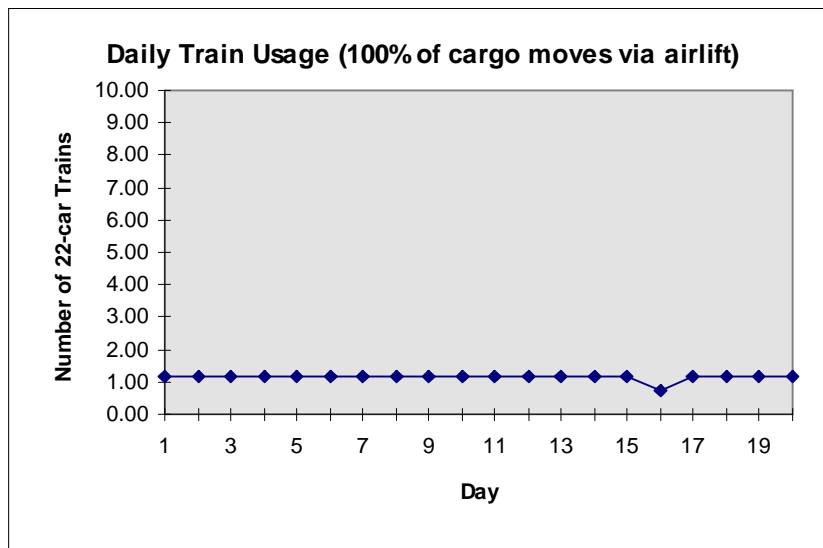
X_t = 56.96 in Figure IV-15.

Figure IV-16 Daily Trains Used - "Must Air" for Cargo

X_r = 1.15 in Figure IV-16.

SOLUTION FOR BUDGET CONSTRAINT OF $4 MILLION:

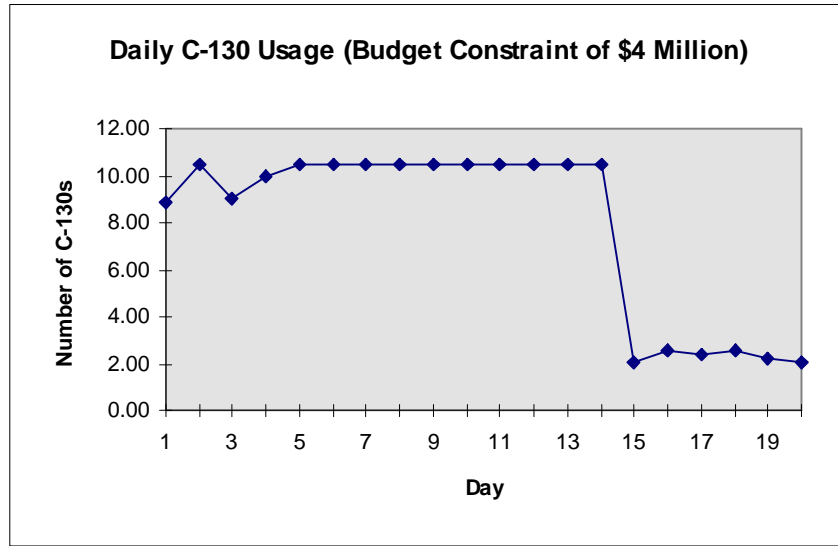**Figure IV-17 Daily C-130s Used - Budget Constraint**

Xp = 10.46 in Figure IV-17. This is the same value as the original problem. Even though $4 million is less than the operating costs required for the solution to the original problem, it was not sufficiently less to cause the maximum C-130s assigned on any given day to become less than the original problem. The total number of C-130 missions over the entire period, however, was in fact less than the original problem.

Another way to force C-130s to carry all the cargo is to set the upper limit for trucks and trains to zero. Of course, C-130s would also get all the passengers and the medical evacaution demands, as well.
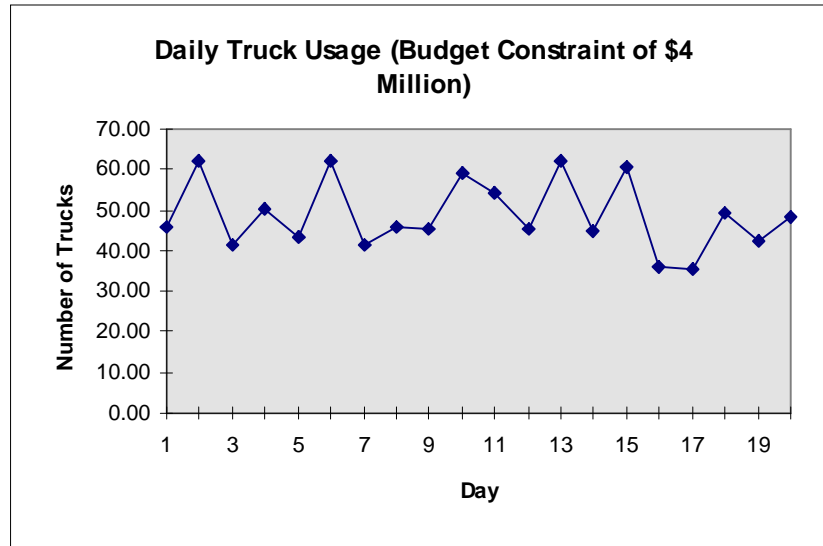
Figure IV-18 Daily Trucks Used - Budget Constraint

$X_t = 62.07$ in Figure IV-18. This is the same as the original problem.
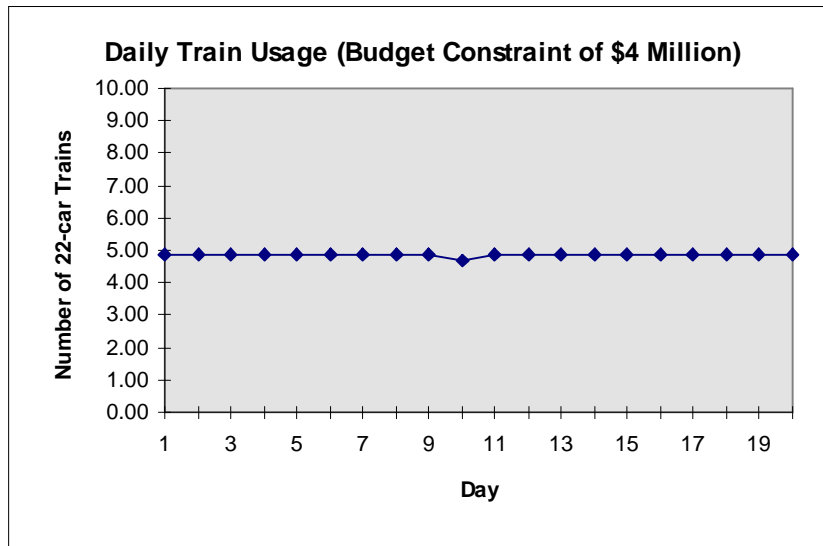

Figure IV-19 Daily Trains Used - Budget Constraint

$X_r = 4.89$ in Figure IV-19. This is the same as the original problem.

SCENARIO OF THE PARAMETRIC ANALYSIS:

The analysis of interest in this demonstration was chosen to be the effect on required transportation assets of moving varying amounts of the load requirement from the first week of the contingency to the second week. A motivation for this analysis was the observation of long delays of all modes of transport going to Bosnia-Herzogovina due to weather. In this vein, the decision maker could have responded with the question, "What amount and mix of transportation is required if the weather forecast suggests we could lose our ability to move equipment and personnel in the first week of the contingency, while maintaining the same closure date?" To conduct this particular analysis, the D-vector or change vector was structured so that cargo, passenger, and medical evacuation requirements would be transferred from the first week to the second week in proportion to the value of D. In other words, as the value of D went from zero to one, a corresponding amount of requirements transferred from the first week to the second week. For example, a D value of 0.51 indicates that 51% of the week one requirements were transferred to week two. The requirements for the other days of the contingency remained the same.
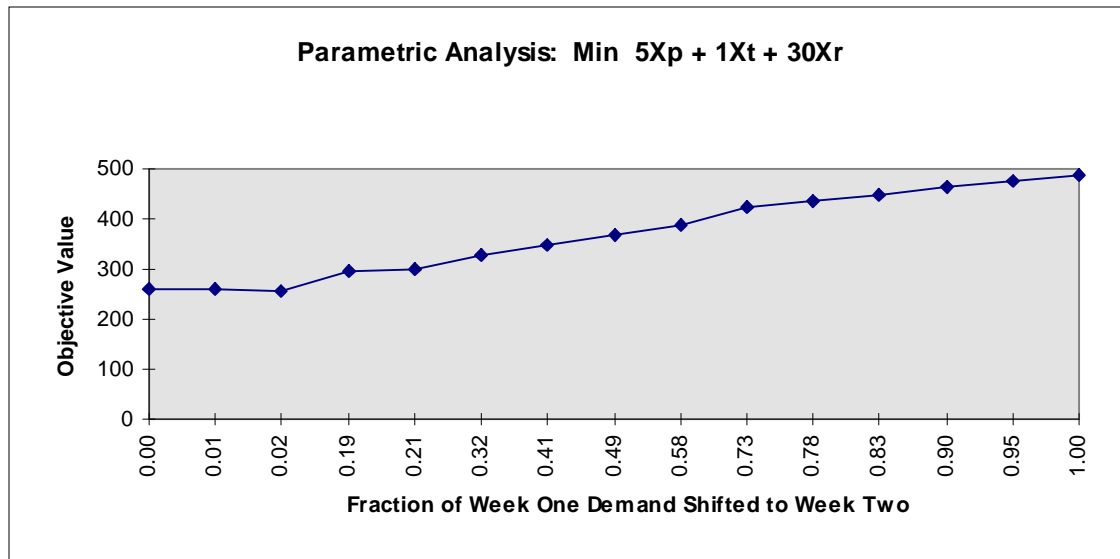
Figure IV-20 Graph: Parametric Analysis of a Potential Scenario

As one would expect, the objective function increases as the cargo, passenger, and medical evacuation requirements are shifted from week one to week two. Since the graph in Figure IV-20 plots the value of the objective function, which in turn is based on the variables Xp, Xt, and Xr, it increases as week two demands increase. This value is based on the maximum number of vehicles used on any given day and the shift in demand from week one to week two insures that week two has the highest demand of any part of the planning horizon.   Consequently, the variables Xp, Xt, and Xr will reach their maximum values in week two. This can be seen in the graph shown in Figure IV-21 below.
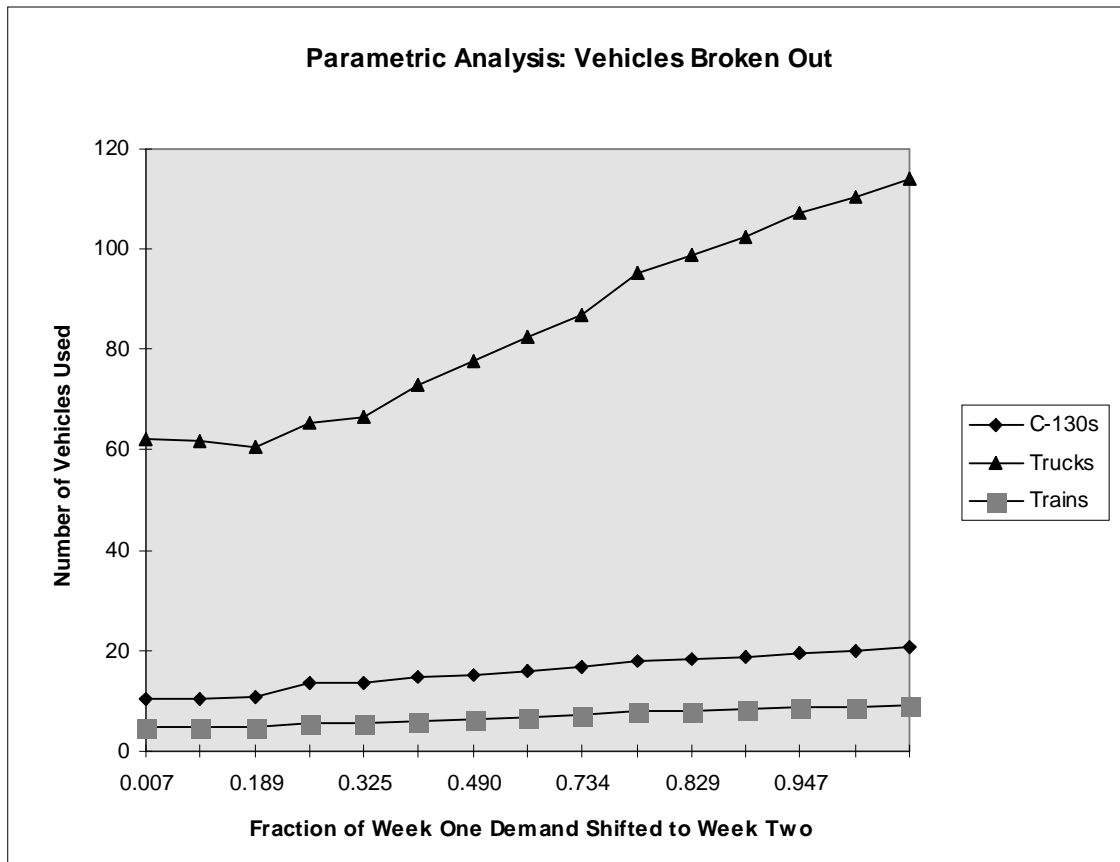
Figure IV-21 Parametric Analysis for a Potential Scenario (Objective removed)

The maximum number of C-130s (Xp), Trucks (Xt), and 22-car trains (Xr) increases as the amount of week one demand is shifted to week two. How this breaks out at various points along the horizontal axis is shown in Figure IV-22 and IV-23. Figure IV-22 shows the assignment of vehicles when 50% of the demand is shifted from week one to week two. Figure IV-23 shows the assignment of vehicles when 100% of the demand is shifted from week one to week two.
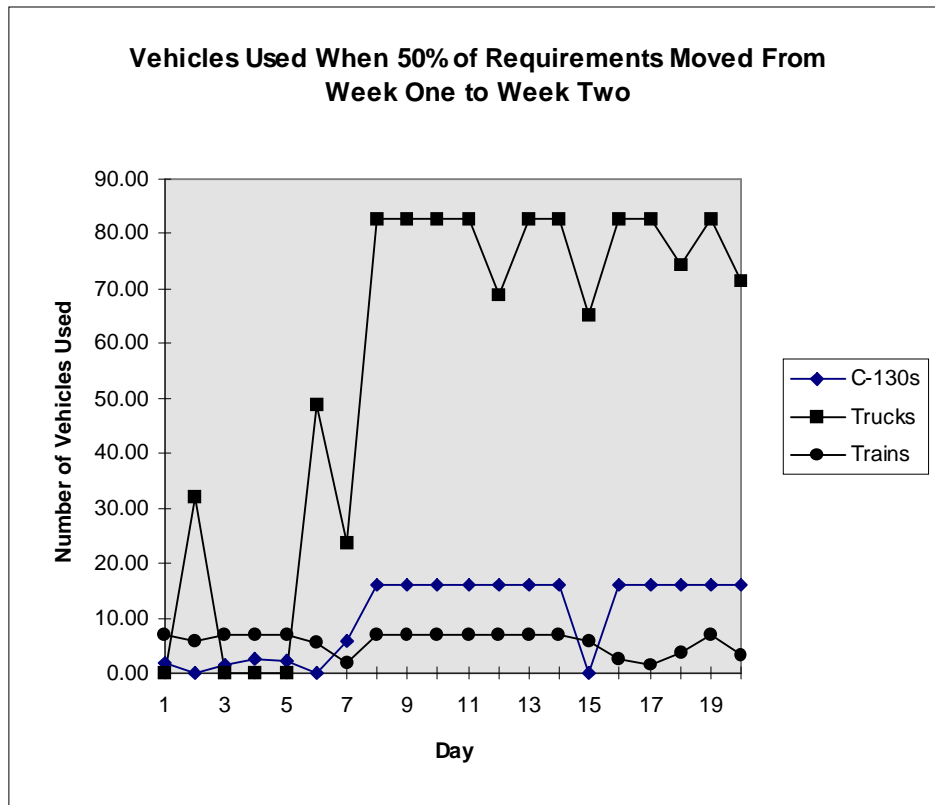
Figure IV-22 Transportation for a 50% Transfer - Week 1 to 2

In Figure IV-22, we see the number of vehicles assigned to week one has decreased when 50% of the demand is shifted to week two.
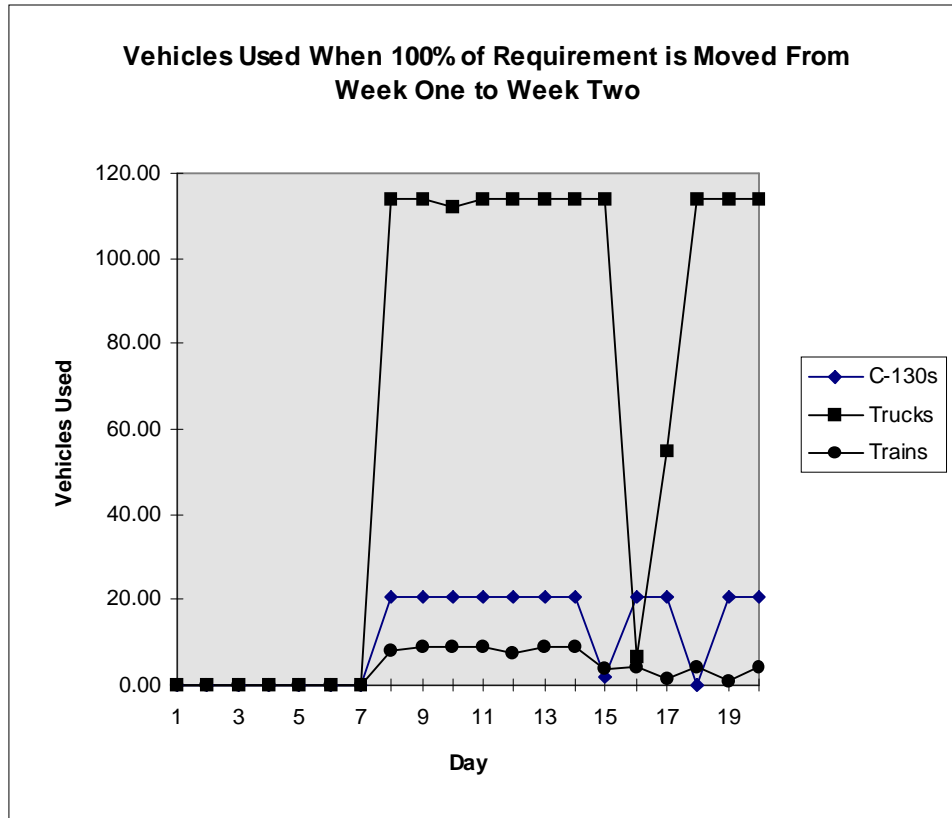
Figure IV-23 Transportation for a 100% Transfer - Week 1 to 2

As expected, the number of vehicles assigned to week one is zero in Figure IV-23 since 100% of week one's demand is moved to week two. The maximum number of vehicles assigned is approximately double the number in the original problem. Consequently, we can say that we require twice the number of vehicles when the demand is shifted from week one to week two, even though the total amount of demand remains unchanged. In order to use the same amount of vehicles as the original problem, we would have to extend the closure date by a week to make up for the week lost due to weather.

V.   Conclusions and Recommendations

The spreadsheet model generated an approximate "ball-park" answer to the question of how many C-130s are required for a given movement requirement.   The chief advantage to this approach is its simplicity and the ease by which the user can set the model up and manipulate the parameters for a parametric analysis. To properly evaluate the answer generated, it was necessary to understand the limitations involved with this particular formulation. As with any mathematical model, reality was not perfectly mapped onto the solution vector. Even the approach outlined by Dr. Edward F. Yang rested on simplifying assumptions, although it captured considerably more detail in its solution process.

One problem of the capacity approach in formulating this problem is that an even capacity is applied in every instance. The reader will readily agree that not all loads will be exactly 25,000 pounds. Since some loads will be necessarily less than the capacity stated in the problem, this formulation may yield an unrealistically small number of vehicles required. An educated "fudge" factor could be applied that was based on empirical data from historical records of the average load size. The user would then alter the number provided by the spreadsheet model accordingly. The reader should understand, however, that the average load size is a random variable and will vary somewhat for every deployment, exercise, and contingency. Only if the same exact units at the same exact force level were deployed in each instance would this number be a constant. Another way to approach this problem would be to try a Monte Carlo

technique for the capacities involved in this problem. Random draws from a probability distribution could be made on the capacities for each vehicle and applied to a statistically valid number of runs in the formulation in Figure III-I. Again, the probability distribution would have to be based on empirical data.

Another problem with this formulation is it does not allow mixed loads. For example, a C-130 might carry several Highly Mobile Military Wheeled Vehicles (HMMWV), a 463L pallet with baggage, and ten passengers. This model would only allow for a full load of cargo, which would be quantified by weight, or a full load of passengers. Combinations of different type loads are not represented.

The assumptions and simplifications of the spreadsheet model are recounted below to allow the reader a better understanding of what the model can and cannot do:

(1) Transportation requirements are aggregated into daily requirements. There probably will be no distinct dividing point in a real contingency, since transportation activities will occur continously twenty-four hours a day and vehicle cycle times may cause vehicle availability, and therefore capacity, to vary from day to day despite a costant fleet size. When working in reverse using this logic to figure a fleet size, this aggregation of daily requirements assumes all required vehicles move that day's requirement within that day and nothing "spills over" into the next day.

(2) The model assigns vehicles in a manner that assumes the entire vehicle capacity is used for each vehicle, i.e., partial loads are not considered among vehicles for a given day's vehicle usage.

(3)   Mixed loads are not considered. Vehicles are either dedicated to cargo movement, passenger movement, or medical evacuation but no combinations between these load types are considered.

(4) Routing of vehicles are not considered. The length or duration of routes could impact the number of vehicles that must be scheduled against a particular transportation requirement. This would depend on the individual routes of each vehicle assigned.

(5) The capacity of onload and offload facilities are ignored. If a facility cannot load quickly enough or if "parking" is too limited, a queue of vehicles could develop. If the system used at the facility for load handling was inadequate, a similar result ensues. Insufficient refuel and maintenance capability also can impact the throughput of a facility. This would represent a bottleneck in the network and could constrain vehicles scheduled as well as place an upper limit on the amount of cargo, passengers, and medical evacuation moved through that facility.

(6) The user must be able to break out the transportation requirements into daily numbers for input into the model.

(7) The user must know the capacities and utilization rates of the vehicles under study.

(8) Vehicle crew scheduling is indirectly considered in the utilization rate. If the crew ratio is low enough, the number of vehicles that can be generated is limited. This will lower the utilization rate. The crew ratio is defined as the total number of crews divided by the total number of vehicles.

The approach that Dr. Edward F. Yang uses alleviates of the first four limitations and makes the problem posed by (6) and (7) a little easier to handle. The model that Dr. Yang developed "reads in" the transportation requirements by directly using the data from an appropriate TPFFD. (Yang, 1995:91) Still, the limitation posed by (5) was not addressed by this model or any others that are known to the author. It should be noted that limitation (5) requires significant additional modeling to be adequately captured and also represents a great deal of uncertainty. The models studied by the author generally handled this aspect of the network implicitly by how other variables were defined. For instance, Dr. Yang handled this in two ways: (1) by assuming the data in the TPFDD accounted for facility throughput and (2) by inserting a service time representing the length of time a vehicle spent at a facility undergoing maintenance, fueling (if required), and loading operations. (Yang, 1995:xv, 23, 67)

An improvement to the spreadsheet model would be to add a Visual Basic module that converts the model formulation into MPS format. The author spent a considerable amount of time manually converting the spreadsheet model into MPS format used to conduct the parameter analysis.

Additional constraints could be added to represent the flows of vehicles. This would constitute including cycle times in the model so that vehicles are made availabe for another mission when finished with a previous mission. This follows the idea of a balance equation, *flow in = flow out*. For example, the following equation can be used (Wing,1991:7-8):

$$\left[\sum_{i \in T(i,j,k)} \sum_l LI(i,j,k,l) \right] + AV(j,k) = AV(j,k-1) + MJO(j,k) + NEW(j,k) +$$

$$\sum_{i \in T(i,j,k-CT(j))} \sum_l LI(i,j,k-CT(j),l) \quad \forall \ j,k$$

LI (i, j, k, l) is a variable representing the lift by lift asset (j) for unit (i) and cargo type (l) on day (k);

AV(j, k) is a variable representing a lift asset (j) available for assignment on day (k);

CT(j) is a variable representing the cycle time (in days) for a lift asset;

MJO(j, k) is a variable representing the initial inventory of lift assets;

NEW(j,k) is a variable representing the new lift required on day (k);

T(i, j, k) are the allowable combinations of i, j, and k for starting lift missions defined by ((k ≥ ALD(i)) and (k ≤ (RDD(i) - HCT(j)) where ALD(i) is the allowable load date for unit (i); RDD(i) is the required delivery date for unit (i); and HCT(j) is the half cycle time for lift asset (j). (Wing, 1991:7-8)

If the user desires a more detailed analysis with less aggregation, a model such as the one cited by Dr. Edward F. Yang (NETO) should be obtained. A model that seems to fall about midway between the spreadsheet model and the NETO model is the Mobility Optimization Model (MOM), which was the source for the constraint listed in the preceding paragraph.

The most necessary improvement to the spreadsheet model of this research is to develop an algorithm to assign appropriate objective function coefficients to the decision variables. The solutions have been shown to be sensitive to the choice of decision

variable weights. If the necessary information could be factored into the decision variable weights, then the objective function would realistically measure decision maker preferences and the priority of one mode of travel versus the others. Then the solution could be used to say that it is the optimal mix of transportation based upon the factors included in the objective function coefficients. This would tell the decision maker the necessary trade-off between one mode of transportation and another. This is a crucial aspect of conducting intratheater airlift analysis. There exist competing modes of transportation that can carry the necessary units and their personnel and equipment within the Korean scenario, so each mode must be considered in turn when justifying the required fleet size for C-130 airlift assets.

VI. APPENDIX A: Summary of Network Optimization Dissertation

The two major problems with an optimization approach to solving the type problem posed by this thesis are 1) complexity and 2) modeling accuracy. (Yang, 1995:14) An example of the complexity issue is the traveling salesman problem (see Chapter II). This problem loosely falls under the heading of a combinatorial optimization problem. If such a problem had many cities to visit, it would be impractical to explicitly enumerate all of the possible combinations and even implicit enumeration with techniques such as "branch and bound" would be computationally expensive. For this reason, heuristic methods, which quickly lead to a "good" solution (but not necessarily optimal) are often used. (Winston, 1994:519-527) The modeling accuracy problem was alluded to previously when the limitations of the spreadsheet model were explained. The real world is inherently complex and possesses factors which are nonlinear and are characterized by varying degrees of uncertainty. (Yang, 1995:14) Notwithstanding these limitations, recent breakthroughs in optimization techniques have made linear programming more broadly applicable, especially in the areas of Mixed Integer Programming and computational aspects. (Yang, 1995:15)

The optimization technique explored in this appendix is based upon the work of Dr. Edward F. Yang's dissertation completed in August 1995. He is essentially modeling the airlift network by specifying every possible route that an aircraft could take and choosing the optimal set of routes. Instead of enumerating each possible route before

optimizing the model, the routes are picked by passing a series of feasibility tests that check for route feasibility, allowable pick up and delivery times, and load compability with the transportation vehicle. To accomplish this, the first task is to transform an operations network into an optimization network. The operations network consists of all the relevant data such as air bases, air routes, onloads, offloads, cargoes, transportation vehicles, scenario, movement requirements, other logistics factors, etc. The optimization network is a labeled digraph suitable for use in mathematical programming. (Yang, 1995:19) To build the optimization network, the digraph topology must be built and labels must be computed. The topology consists of nodes in the network that correspond to customers requiring onload and offload. The labels consist of information such as the cost of an arc between nodes, the required delivery or pickup time for a cargo load, and all of the physical nodes that make up the arc. (Yang, 1995:17-21)

The optimization network is represented by the notation, $G(N, \Lambda)$. The symbol "N" refers to the nodes or locations (air base, terminal, etc.) in the network and the symbol "Λ" refers to the arcs or routes between the nodes in the network. There are four types of nodes in "N". The symbol "S" represents starting nodes or origination points for a given set of vehicles. The symbol "P^+" represents pickup or onload nodes, the symbol "P^-" represents delivery or offload nodes and the symbol "T" represents ending or termination nodes. An arc between two sets of nodes is represented by the symbol "x", i.e., the arcs between the pickup nodes and the delivery nodes is symbolized by "P^+ x P^-". A complete digraph is formed by P^+ x P^-. Only S arcs go from S to P^+ and only T arcs

go from P^- to T. The symbol "P" represents all the pickup and delivery nodes, i.e., $P = P^+$

x P^-. In like manner, $N = S \cup P \cup T$ and $\Lambda = S \times P^+ \cup P^+ \times P^- \cup T$.   (Yang, 1995:19)
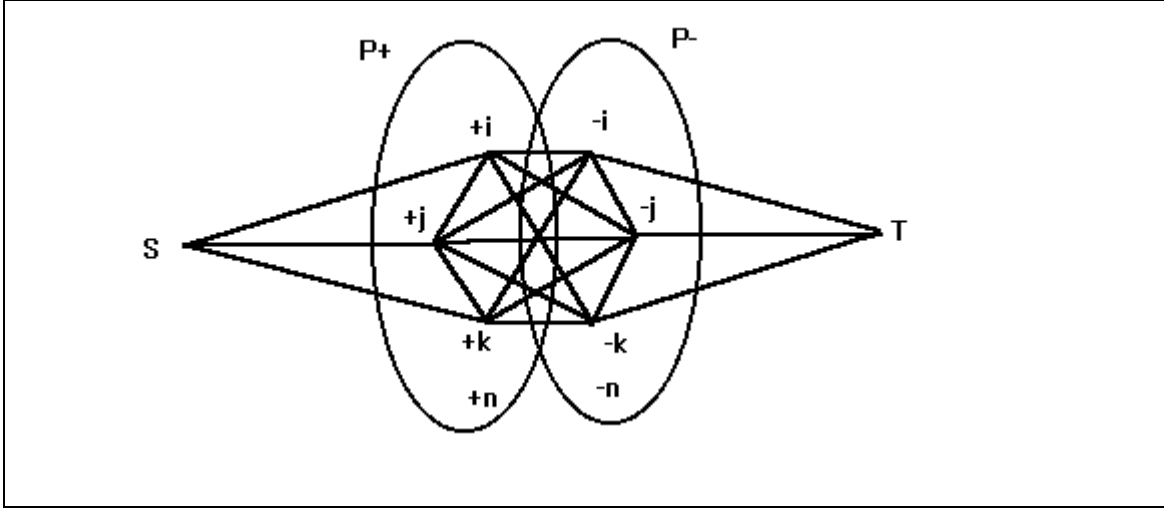

Figure VI-1 The Optimization Network (Yang, 1995:22)

Figure VI-1 shows a pictorial representation of a generic optimization network, $G(N, \Lambda)$. Within the pickup nodes, P^+, and the delivery nodes, P^-, there are a total of n customers (or requirements) indexed by i. A pickup location of a customer i is associated with a node i and the respective delivery location is associated with a node $n+i$. The node i can also be designated by the symbol "$+i$" and the node $n+i$ can be designated with the symbol "$-i$" as in Figure VI-1. The starting node is designated as node 0 and the termination node is designated by node 2n+1, i.e., $S = \{0\}$ and $T = \{2n+1\}$.   Therefore, $N = \{0, 1, \ldots, n+1, \ldots, 2n, 2n+1\}$ and the node set $P = P^+ \cup P^-$ is made up of the pickup node set such that $P^+ = \{1, 2, \ldots, n\}$ and the delivery node set such that $P^- = \{n+1, n+2, \ldots, 2n\}$. (Yang, 1995:21-22)

The remaining labeling information is found from load information, vehicle information, and vehicle capacity information. A customer i will demand that $\bar{d_i}$ units be moved from node i to node $n+i$. A pickup time window can be established for node i and is denoted by $[a_i, b_i]$, where a_i is the earliest time a load can be picked up and b_i is the latest time a load can be picked up. Similarly, a delivery time window is described by the notation $[a_{n+i}, b_{n+i}]$. The window $[a_0, b_0]$ is the time window for the vehicle's departure from the departure base, node S, and $[a_{2n+1}, b_{2n+1}]$ is the time window for the vehicle's arrival back at the recovery base, node T. The set of vehicles made available to move the load requirements is represented by "V" where V = { 1, 2, . . ., |V|}. These vehicles are indexed by the symbol "v". The symbol $\bar{D}$ represents the capacity of each vehicle. (Yang, 1995:21-22)

The problem is modeled as a basic vehicle routing problem (VRP). This approach uses the Set-Partition Model to formulate the VRP. Consider Ω the set of all feasible routes r, and let δ_{ir} be a binary coefficient such that:

$$\delta_{ir} = \begin{cases} 0: & node\ i\ is\ not\ on\ route\ r \\ 1: & node\ i\ is\ on\ route\ r \end{cases}$$ Let c_r^* be the optimal cost of route r and x_r, a

binary variable such that $x_r = \begin{cases} 0: & route\ r\ is\ not\ picked \\ 1: & route\ r\ is\ picked \end{cases}$ in the optimal solution.

The problem is then formulated as in Figure VI-2.

$$\text{minimize} \quad \sum_{r \in \Omega} c_r^* x_r$$

$$\text{subject to} \quad \sum_{r \in \Omega} \delta_{ir} x_r = 1 \quad (i \in V\backslash\{0\})$$

$$x_r \in \{1,0\} \quad (r \in \Omega)$$

Figure VI-2 General Set-Partition Model (Yang, 1995:51)

In this manner, each column can be thought of as one possible route and each row can be thought of as associated with a vertex or node excluding the node of origination, 0. The symbol "V" represents the set of vertexes in figure IX-2 and is equivalent to the symbol "N" mentioned earlier. Elsewhere, it will represent the set of vehicles as described previously. (Yang, 1995:51)

With some minor adjustments, the set-partition formulation in Figure VI-1 can be recast for the digraph that was defined as $G(N, \Lambda)$.   The column coefficients are represented by the symbol "δ_r" and are defined as: $\delta_r = [\delta_{ir}]_{nx1}$,   where

$$\delta_{ir} = \begin{cases} 0: & \textit{if node } i \textit{ is not on route } \rho_r \\ 1: & \textit{if node } i \textit{ is on route } \rho_r \end{cases} \quad i \in P^+ .$$

The symbol "ρ_r" represents a feasible route in $G(N, \Lambda)$. It is a non-cyclic path that originates from S and terminates at T while satisfying pairing constraints, precedence constraints, capacity constraints, and time window constraints. (Yang, 1995:66) If we define x_{ij} as a binary route flow variable such that:

$$x_{ij} = \begin{cases} 1 & \textit{if the feasible route } r \textit{ goes directly from } i \textit{ to } j \\ 0 & \textit{if the feasible route } r \textit{ does not go directly from } i \textit{ to } j \end{cases} \quad (i, j) \in \Lambda$$

then we can define ρ_r with the formulation in Figure VI-4 (Yang, 1995:67)::

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{j,n+i} = 0, \quad i \in P^+ \qquad \text{(pairing constraints)}$$

$$T_i + s_i + t_{i,\,n+i} \le T_{n+i}, \qquad i \in P^+ \qquad \text{(precedence constraints)}$$

$$RT: \quad \left.\begin{aligned}
x_{ij} = 1 &\Rightarrow T_i + s_i + t_{ij} \le T_j, \ \ i, j \in P \\
x_{0j} = 1 &\Rightarrow T_0 + t_0 \le T_j, \ \ j \in P^+ \\
x_{i,2n+1} = 1 &\Rightarrow T_i + s_i + t_{i,2n+1} \le T_{2n+1}, \ i, j \in P
\end{aligned}\right\} \qquad \text{(time progression)}$$

$$\left.\begin{aligned}
a_i \le T_i &\le b_i, \ \ i \in P \\
a_0 \le T_0 &\le b_0 \\
a_{2n+1} \le T_{2n+1} &\le b_{2n+1}
\end{aligned}\right\} \qquad \text{(time window constraints)}$$

$$\left.\begin{aligned}
x_{ij} = 1 &\Rightarrow \overline{Y}_i + \overline{d}_j = \overline{Y}_j, \ \ i \in P, \ j \in P^+ \\
x_{ij} = 1 &\Rightarrow \overline{Y}_i - \overline{d}_j = \overline{Y}_j, \ \ i \in P, \ j \in P^- \\
x_{0j} = 1 &\Rightarrow \overline{Y}_0 + \overline{d}_j = \overline{Y}_j, \ \ j \in P^+
\end{aligned}\right\} \qquad \text{(load progression)}$$

$$0 \le \overline{Y}_i \le \overline{D}, \quad i \in P^+ \qquad \text{(capacity constraints)}$$

Figure VI-3 Feasible Route Formulation (Yang, 1995:67)

Now we can redefine the variable x_r found in Figure VI-2 based upon the formulation of a feasible route in Figure VI-3. Then

$$x_r = \begin{cases} 0: & \textit{if the feasible route } \rho_r \textit{ is not selected in the solution} \\ 1: & \textit{if the feasible route } \rho_r \textit{ is selected in the solution} \end{cases}, \quad r \in \{r|\, \rho_r \in \Omega\}$$

In this version of the set-partitioning formulation, the column coefficients, δ_r, and the cost coefficients, c_r, are not explicitly available. They must be obtained by finding the corresponding feasible routes through the formulation in Figure VI-3. (Yang, 1995:68) The column coefficients are found by a column generating algorithm that is constructed

from a shortest path problem with constraints. It basically solves the formulation in Figure VI-3. (Yang, 1995:72) The cost coefficients are found by the equation $c_r = \sum\limits_{(i,j)\in\Lambda} c_{ij} x_{ij}$.

The manner in which we define c_{ij} determines the nature of the problem. For the purposes of this study, this component of the cost coefficient will be defined in a way that causes the objective function to minimize the number of vehicles used. In this case,

$$c_{ij} = \begin{cases} t_{ij} + K \ if \ i = 0 \\ t_{ij} \ \ if \ i \neq 0 \end{cases}$$ where some constant K is added for the portion of the route, x_{0j} .

In a similar manner that was used for the spreadsheet model, a limiting constraint can be added to the model to ensure the total number of vehicles used will not exceed some number, m. The constraint would look like $\sum\limits_{r\in\Omega} x_r \leq m$. As one can imagine, the number of feasible routes, and hence, the number of columns δ_r in $|\Omega|$ can be very large. This is why a column generation method is preferable than enumerating all the feasible routes and solving the set-partitioning problem to integer optimality. (Yang, 1995:68)

Dr. Yang used a modified version of the Bellman-Ford Dynamic Programming Algorithm to solve a Constrained Shortest Path Problem. This is the algorithm which generates the columns for the set-partitioning formulation described above, δ_r . This is a method which expands from node to node while meeting a series of feasibility tests that check for the constraints in Figure VI-3. In order to understand the tests in the algorithm, a few variables and parameters must be defined:

$\rho_\alpha^k (j)$:   the αth route in all routes that start at S and end at *j* with *k* arcs.

$P^k (j)$:   the set of all routes that start at S and end at j with k arcs, so that

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$$P^k(j) = \bigcup_\alpha \rho_\alpha^k(j)$$

$h_\alpha^k(j):$ the cost of route $\rho_\alpha^k(j)$

$T_\alpha^k(j):$ the arrival time at j of route $\rho_\alpha^k(j)$

$\overline{Y}_\alpha^k(j):$ the vehicle load at node j along route $\rho_\alpha^k(j)$.

Stage k is defined as a state in which any of the paths constructed at this point have exactly k arcs from the optimization network G(N, Λ). As depicted in figure IX-4, node i is expanded from stage k-1 to stage k to reach node j. The route $\rho_\beta^{k-1}(i)$ and cost $h_\beta^{k-1}(i)$ lead to node i and node i is expanded to node j with an additional cost of $\tilde{c}_{ij}$ to get $\rho_\alpha^k(j)$ if certain feasibility tests are satisfied. (Yang, 1995:81)
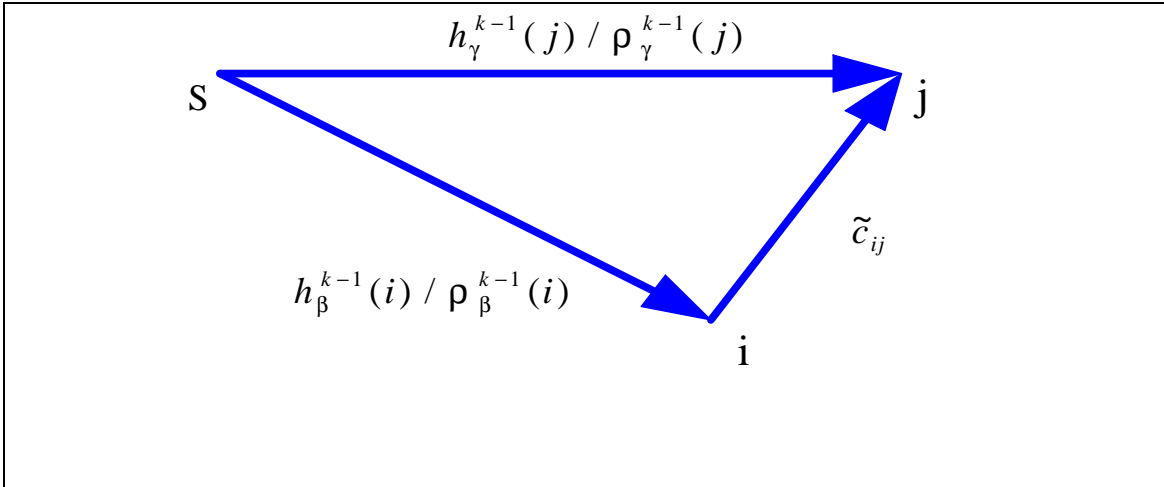

Figure VI-4 Stage k-1 to k   (Yang, 1995:82)

The feasibility tests are as follows:

For pairing and precedence constraints

     if $j \in P^+$ and $j \in \rho_\beta^{k-1}(i)$, then the expansion from i to j for $\rho_\alpha^k(j)$ is not feasible;

if $j \in P^-$, and $j - n \notin \rho_\beta^{k-1}(i)$, then the expansion for $\rho_\alpha^k(j)$ is not feasible;

For time window constraints

$$T_\alpha^k(j) = \max\{a_j, \ T_\beta^{k-1}(i) + s_i + t_{ij}\}$$

if $T_\alpha^k(j) > b_j$ then the expansion to $\rho_\alpha^k(j)$ is not feasible;

For capacity constraints:

$$\overline{Y}_\alpha^k(j) = \begin{cases} \overline{Y}_\beta^{k-1}(i) + \overline{d}_j, & j \in P^+ \\ \overline{Y}_\beta^{k-1}(i) - \overline{d}_j, & j \in P^- \end{cases}$$

if $\overline{Y}_\alpha^k(j) > \overline{D}$, then the expansion to $\rho_\alpha^k(j)$ is not feasible; (Yang, 1995:82)

Otherwise, the expansion from i to j is made and $\rho_\alpha^k(j)$ is constructed as follows:

$$\rho_\alpha^k(j) = \rho_\beta^{k-1}(i) \cup \{j\}$$

$$P^k(j) = P^k(j) \cup \rho_\alpha^k(j)$$

$$h_\alpha^k(j) = h_\beta^{k-1}(i) + \tilde{c}_{ij} \qquad \text{(Yang, 1995:83)}$$

for k = 1, 2n+1 (for each stage k)

for each node $j \in P \cup T$

for each node $i \in \delta^-(j)$

for each β of route $\rho_\beta^{k-1}(i) \in P^{k-1}(i)$
Test Feasibility
Store the new route $\rho_\alpha^k(i)$
next β

next i

next j
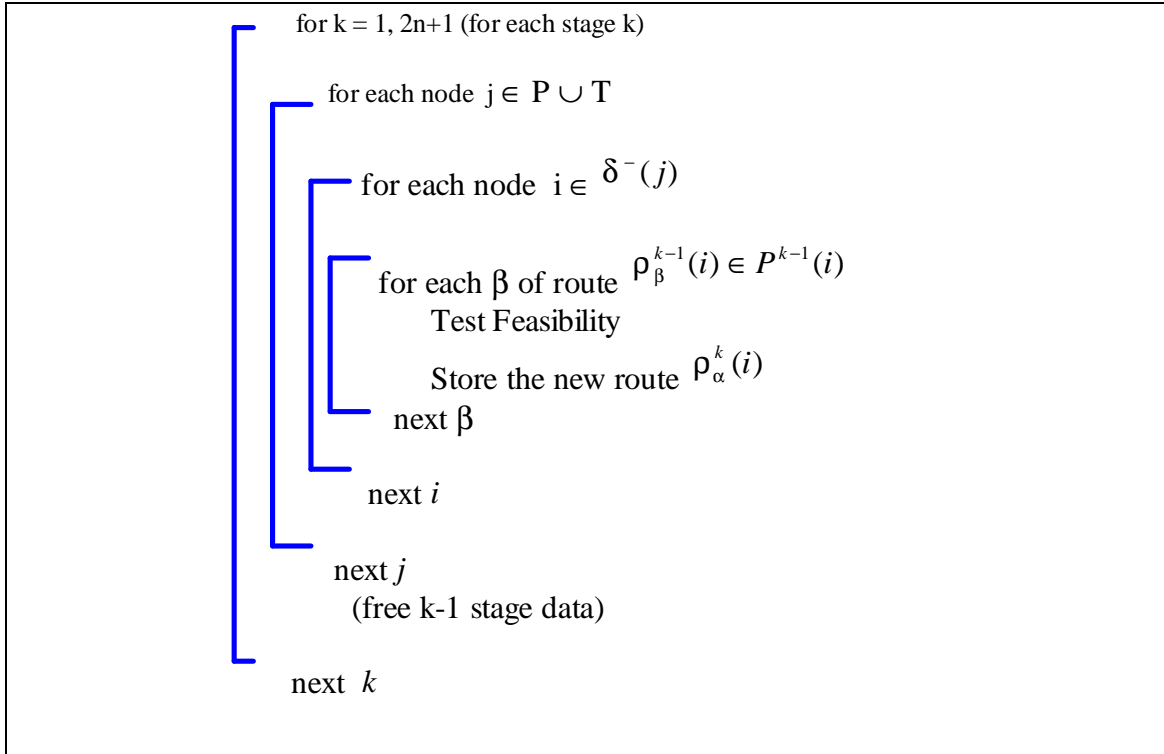(free k-1 stage data)

next k

Figure VI-5 Constrained SPP Algorithm (Yang, 1995:83)

Preprocessing can be conducted to eliminate any potential infeasible or inferior paths as a way to keep the number of stored paths as small as possible. For a specific node (except for node T) all feasible k-arc paths at stage k should be stored. A present feasible path could become infeasible at later stages. At node T, either the shortest path or the p shortest paths can be stored. When constructing a path with k arcs, only the paths with k-1 arcs are involved. Paths with a different number of arcs do not need to be stored. Store the paths with k-1 arcs only. (Yang, 1995:84)

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VIII. Vita

Capt Philip B. Oglesby was born 17 March 1961 in Bend, Oregon. He graduated from William B. Travis High School in 1979 and entered undergraduate studies at the University of Texas at Austin, Texas. After completing two semesters at the University of Texas, he entered undergraduate studies at the United States Air Force Academy in Colorado Springs, Colorado. He graduated with a Bachelor of Science degree in Russian Studies in May 1984. He received his commission on 30 May 1984 upon graduation from the Academy. His first assignment was at Norton AFB as a C-141B tactical airdrop navigator. His second assignment was at Little Rock AFB as a C-130E tactical airdrop navigator and as a Joint Airlift Exercise Planner for the Joint Readiness Training Center at Fort Polk, Louisiana. In August 1994, he entered the School of Engineering, Air Force Institute of Technology.

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